Unification Grammars and Off-Line Parsability

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Research Thesis

Submitted in partial fulfillment of the requirements
for the degree of Master of Science in Computer Science

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Submitted to the Senate of
the Technion — Israel Institute of Technology

Kislev 5763 Haifa November 2002

The research was done under the supervision of Prof. Nissim Francez and Dr. Shuly Wintner in the department of Computer Science.

I especially would like to thank **Shuly** and **Nissim** for their excellent guidance and support. They were a great source of inspiration and I have learned a lot from them.

I would like to thank Prof. Kaminski for all his administrative support while Nissim was abroad.

The generous financial help of the Technion is gratefully acknowledged.

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Abstract

Context-free grammars are considered to lack the expressive power needed for modeling natural languages. Unification grammars have originated as an extension of context-free grammars, the basic idea being to augment the context-free rules with feature structures in order to express additional information. Unification grammars are known to be Turing-equivalent; given a grammar G and a word w, it is undecidable whether there exists a derivation tree for w admitted by G.

In order to ensure the decidability of the membership problem, several constraints on grammars, commonly known as the *off-line parsability constraint (OLP)* were suggested. The membership problem is decidable for grammars which satisfy OLP. An open question is whether it is decidable if a given grammar satisfies OLP.

In this thesis we investigate various definitions of OLP and discuss their inter-relations. As some of the OLP variants were suggested without recognizing the existence of all other variants, we make a comparative analysis of the several different OLP variants for the first time. Some of the OLP variants were conjectured to be undecidable (it is undecidable whether a grammar satisfies the constraint), although no proof has ever been provided. There exist some variants of OLP for which decidability holds, but these conditions are too restrictive: there is a large class of non-OLP grammars for which parsing termination is guaranteed. Also these variants are limited only to a specific kind of unification grammar formalisms.

Our main contribution is proofs of undecidability for three of the undecidable OLP variants and a novel OLP constraint, along with an algorithm for deciding whether a grammar satisfies it. Our constraint is applicable to all unification grammar formalisms. It is more liberal than the existing decidable constraints, yet it can be tested efficiently.

Notation and Abbreviations

OLP	—	Off-line parsability.
$\mathbf{OLP_{PW}}$	—	Pereira and Warren's OLP.
$\mathbf{OLP_{JO}}$	—	Johnson's OLP.
$\mathbf{OLP_{S}}$	—	Shieber's OLP.
$\mathbf{OLP_{D}}$	—	A novel decidable OLP constraint.
DB	—	Depth-boundedness.
FA	—	Finite ambiguity.
HP	—	Honest Parsability.
LFG	—	Lexical functional grammars.
DCG	—	Definite clause grammars.

Chapter 1

Introduction

1.1 Background and literature survey

Context-free grammars are considered to lack the expressive power for modeling natural languages. Unification grammars have originated as an extension of context-free grammars, the basic idea being to augment the context-free rules with non context-free annotations (feature structures) in order to express some additional information, such that the constraints of the grammar will be sensitive to these features. Unification grammars have the ability to describe phonological, morphological, syntactic and semantic properties of languages and thus they are linguistically plausible for modeling natural languages. They have a stronger generative capacity than context-free grammars; a unification grammar may generate an infinite set of feature structures and thus there exist no mapping of the infinite set to the finite set of categories of a context-free grammar.

Today, several formalisms of unification grammars exist, some of which do not necessarily assume an explicit context-free backbone. Among the unification grammar formalisms are Definite Clause Grammar, DCG (Pereira and Warren, 1980), Lexical Functional Grammar, LFG (Kaplan and Bresnan, 1982), Generalized Phrase Structure Grammar, GPSG (Gazdar et al., 1985), Head-Driven Phrase Structure Grammar, HPSG (Pollard and Sag, 1986), PATR-II (Shieber, 1986) and many others.

The **recognition problem** (also known as the membership problem), for a grammar G and a string w, is whether $w \in L(G)$. The **parsing problem**, for a grammar G and a string w, is to generate all parse trees that G induces on w, determining what structural descriptions are assigned by G to w. Unfortunately unification grammars are Turing equivalent (i.e., RE) in their generative capacity. Determining whether a given string is generated by a given grammar is equivalent to deciding whether a Turing machine halts on the empty input (Johnson, 1988). Therefore, the recognition/membership problem for unification grammars is undecidable in the general case.

In order to guarantee decidability of the recognition problem, several constraints on grammars, commonly known as the *off-line parsability constraint (OLP)* were suggested. The recognition problem is decidable for OLP grammars. Several variants of OLP were suggested (Pereira and Warren, 1983; Johnson, 1988; Haas, 1989; Torenvliet and Trautwein, 1995; Shieber, 1992; Francez and Wintner, 1999; Kuhn, 1999). Some variants are applicable only to skeletal grammar formalisms, while others are applicable to all unification grammar formalisms. The main open question is, whether it can be determined if a grammar satisfies OLP.

1.2 Research objectives

The OLP constraint is a restriction on grammars which guarantees decidability of the recognition problem. Several variants of the OLP constraint are known, some of which were suggested without recognizing the existence of all other variants. In this work we make, for the first time, a comparative analysis of the several different OLP variants. We explore these conditions, discuss their properties, examine the differences and the relationships between these constraints and define a hierarchy among them.

It is well known that for OLP grammars decidability of the recognition problem is guaranteed, but can it be determined whether a grammar satisfies OLP? Some researchers (Haas, 1989; Torenvliet and Trautwein, 1995) conjecture that some of the OLP variants are undecidable (it is undecidable whether a grammar satisfies the constraint), but there exists no proof of it. There exist some variants of OLP for which decidability holds, but these conditions are too restrictive; there is a large class of non-OLP grammars for which parsing termination is guaranteed, furthermore they are limited only to unification grammar formalisms which assume an explicit context-free backbone. In this work we explore these conjectures and provide undecidability proofs for three of the undecidable OLP variants.

Our major contribution is providing a novel decidable OLP constraint which is more liberal than the existing decidable constraints, applies to all unification grammar formalisms, and yet can be tested efficiently. We also provide an algorithm for deciding whether a grammar satisfies the constraint as well as an analysis of the relationships between the novel constraint and the existing OLP variants.

1.3 Structure of this thesis

Chapter 2 presents a thorough introduction to unification grammars including the notations used in this work. Chapter 3 presents a literature survey of several different OLP variants along with a discussion of the properties of each variant. Chapter 4 provides a comparative analysis of the several OLP definitions and the inter-relations among them. Chapter 5 presents proofs of undecidability for three of the OLP variants. Chapter 6 describes our major achievement, a novel OLP constraint. Conclusions and suggestions for further research are given in Chapter 7.

Chapter 2

Unification grammars

Many modern linguistic theories use *feature structures* to describe linguistic objects representing phonological, morphological, syntactic, and semantic properties of natural languages. These feature structures are specified in terms of constraints and are used to represent lexical items, phrases and rules. Unification is the primary operation for determining the satisfiability of conjunctions of constraints. Unification is based on *subsumption* of feature structures, where subsumption is a partial pre-order on feature structures which expresses whether two feature structures are consistent and whether one structure contains more specific information than the other.

Unification grammar formalisms are based on unification of feature structures. The unification operation takes two feature structures and combines the information contained in them to produce a feature structure that includes all the shared information of both. If they contain incompatible information the unification operation fails. Non-failing unification returns the least upper bound *(lub)* of its arguments (with respect to the subsumption ordering).

A detailed description of unification grammars is given by Shieber (1986; 1992) and Carpenter (1992). In this chapter we present the grammar formalism used in this work. Presentation and notation are based on Francez and Wintner (In preperation).

2.1 Preliminaries

The following definitions are based on Carpenter (1992) and Francez and Wintner (1999) and will be used later for defining general and skeletal unification grammars. Both grammar formalisms are defined over a finite set FEATS of features and a finite set ATOMS of atomic values. Skeletal grammars are defined over an additional parameter, a finite set CATS of categories.

Definition 2.1. *A **feature structure** $fs = \langle Q, \bar{q}, \delta, \theta \rangle$ is a directed, connected, labeled, rooted graph consisting of a finite, nonempty set of nodes Q, a root $\bar{q} \in Q$, a partial function, $\delta : Q \times \text{FEATS} \to Q$ specifying the arcs, such that every node $q \in Q$ is accessible from $\bar{q}$, and a partial function, marking some of the sinks, $\theta : Q_s \to \text{ATOMS}$, where $Q_s = \{q \in Q \mid \delta(q, f)\uparrow$ for every $f \in \text{FEATS}\}$ (the nodes for which δ is undefined for all features). Meta-variables A, B (with or without subscripts) range over feature structures.*

We use a notation of *attribute-value matrices* (AVMs) to represent feature structures. Such matrices list, within square brackets, a set of features, along with their values. Each row in an AVM

is a pair $\text{F} : v$, denoting that the feature F has the value v; the value can either be atomic or complex, in the form of another AVM. Figure 2.1 depicts an example feature structure, $A = \langle Q, \bar{q}, \delta, \theta \rangle$, represented both as an AVM and as a graph, where $Q = \{q_0, q_1, q_2, q_3\}, \bar{q} = q_0, \delta(q_0, \text{AGR}) = q_1, \delta(q_1, \text{NUM}) = q_2, \delta(q_1, \text{PERS}) = q_3, Q_S = \{q_2, q_3\}, \theta(q_2) = pl, \theta(q_3) = third$.

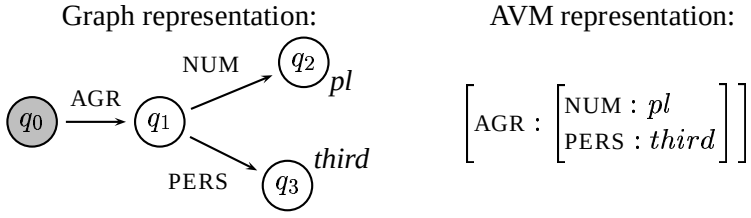


Figure 2.1: A feature structure

Definition 2.2. *A **multi-rooted feature structure** (MRS) is a pair $\langle \bar{Q}, G \rangle$ where:*

- *$G = \langle Q, \delta, \theta \rangle$ is a finite directed, labeled graph consisting of a non-empty, finite set Q of nodes, a partial function, $\delta : Q \times \text{FEATS} \rightarrow Q$, specifying the arcs and a partial function, $\theta : Q \rightarrow \text{ATOMS}$, labeling the sinks.*
- *$\bar{Q}$ is an ordered set of distinguished nodes in Q called **roots**.*

*G is not necessarily connected, but the union of all the nodes reachable from all the roots in$\bar{Q}$ is required to yield exactly Q. The **length** of an MRS is the number of its roots, $|\bar{Q}|$. λ denotes the empty MRS, where $Q = \emptyset$.*

Meta-variables σ, ρ range over MRSs. If $\sigma = \langle \bar{Q}, G \rangle$ is an MRS, and $\bar{q}_i$ is a root in $\bar{Q}$ then $\bar{q}_i$ naturally induces a feature structure $A_i = \langle Q_i, \bar{q}_i, \delta_i, \theta_i \rangle$, where Q_i is the set of nodes reachable from $\bar{q}_i$, $\delta_i = \delta|_{Q_i}$ and $\theta_i = \theta|_{Q_i}$. Thus, σ can be viewed as an ordered sequence $\langle A_1, \ldots, A_n \rangle$ of (not necessarily disjoint) feature structures.

The **sub-structure** of $\sigma = \langle A_1, \ldots, A_n \rangle$, induced by the pair $\langle i, j \rangle$ and denoted $\sigma^{i \ldots j}$, is $\langle A_i, \ldots, A_j \rangle$. If $i > j$, $\sigma^{i \ldots j} = \lambda$. If $i = j$, σ^i is used for $\sigma^{i \ldots i}$.

Definition 2.3. *A **path** is a finite sequence of features; ϵ is the empty path, and we use π (with or without subscripts) to range over paths. The definition of δ is extended to paths in the natural way: $\delta(q, \epsilon) = q$ and $\delta(q, f\pi) = \delta(\delta(q, f), \pi)$.*

Definition 2.4. *An MRS is **reentrant** iff either of the conditions below hold:*

- *there exist two different paths $\pi_1, \pi_2 \in \text{FEATS}^*$, and a root $\bar{q}_i \in \bar{Q}$ such that $\delta(\bar{q}_i, \pi_1) = \delta(\bar{q}_i, \pi_2)$.*
- *there exist two paths $\pi_1, \pi_2 \in \text{FEATS}^*$ (not necessarily disjoint) and two different roots $\bar{q}_i, \bar{q}_j \in \bar{Q}$ such that $\delta(\bar{q}_i, \pi_1) = \delta(\bar{q}_j, \pi_2)$.*

Definition 2.5. *An MRS $\sigma = \langle \bar{Q}, G \rangle$ **subsumes** an MRS $\sigma' = \langle \bar{Q}', G' \rangle$ (denoted by $\sigma \sqsubseteq \sigma'$) if $|\bar{Q}| = |\bar{Q}'|$ and there exists a total function $h : Q \rightarrow Q'$ such that:*

- *for every root* $\bar{q}_i \in \bar{Q}, h(\bar{q}_i) = \bar{q}'_i$
- *for every* $q \in Q$ *and* $f \in$ FEATS*, if* $\delta(q, f)\downarrow$ *then* $h(\delta(q, f)) = \delta'(h(q), f)$
- *for every* $q \in Q$ *if* $\theta(q)\downarrow$ *then* $\theta(q) = \theta'(h(q))$

Definition 2.6. *Let* A, B *be two feature structures. The* ***unification*** *of* A *and* B*, denoted by* $A \sqcup B$ *is defined iff there exists a feature structure* C *such that* $A \sqsubseteq C$ *and* $B \sqsubseteq C$*, in which case it is the most general feature structure* C' *such that* $A \sqsubseteq C'$ *and* $B \sqsubseteq C'$*.*

Unification in context Let σ, ρ be two MRSs. The unification of the i-th element in σ with the j-th element in ρ, denoted by $(\sigma, i) \sqcup (\rho, j)$, is defined iff σ^i is unifiable with ρ^j, in which case it is a pair of MRSs, $\langle \sigma', \rho' \rangle$, where $\sigma'^i = \rho'^j = \sigma^i \sqcup \rho^j$, and when some variable X in σ^i is unifiable with some variable Y in ρ^j, all occurrences of X in σ and of Y in ρ are modified: they are all set to the unified value.

2.2 General unification grammars

Unification grammars have originated as an extension of context-free grammars, although not all unification grammar formalisms assume the existence of a context-free backbone. We refer to those who do as **skeletal**, whereas others are referred to as **general** unification grammars.

Definition 2.7. *A* ***general unification grammar*** *(over* FEATS *and* ATOMS*) is a tuple* $G = \langle \mathcal{R}, \mathcal{L}, A^s \rangle$ *where:*

- $\mathcal{R}$ *is a finite set of rules, each of which is an MRS of length* $n \geq 1$*, with a designated first element, the* ***head*** *of the rule, followed by its* ***body****. The head and body are separated by an arrow (*$\rightarrow$*).*
- $\mathcal{L}$ *is a lexicon, which associates with every terminal* a *(over a fixed finite set* Σ *of terminal words) a finite set of feature structures,* $\mathcal{L}(a)$*.*
- A^s *is the start symbol (a feature structure).*

Definition 2.8 (Derivation). *An MRS* $\sigma_A = \langle A_1, \ldots, A_k \rangle$ ***immediately derives*** *an MRS* $\sigma_B = \langle B_1, \ldots, B_m \rangle$ *(denoted* $\sigma_A \rightarrow \sigma_B$*) iff there exist a rule* $\rho' \in \mathcal{R}$ *of length* n *and an MRS* ρ*,* $\rho' \sqsubseteq \rho$*, such that:*

- $m = k + n - 2$*;*
- ρ*'s head is some element* i *of* σ_A*:* $\rho^1 = \sigma_A^i$*;*
- ρ*'s body is a sub-structure of* σ_B*:* $\rho^{2\ldots n} = \sigma_B^{i\ldots i+n-2}$*;*
- *The first* $i - 1$ *elements of* σ_A *and* σ_B *are identical:* $\sigma_A^{1\ldots i-1} = \sigma_B^{1\ldots i-1}$*;*
- *The last* $k - i$ *elements of* σ_A *and* σ_B *are identical:* $\sigma_A^{i+1\ldots k} = \sigma_B^{m-(k-i+1)\ldots m}$*.*

The reflexive transitive closure of '$\rightarrow$' is denoted '$\overset{*}{\rightarrow}$'.

An MRS σ **derives** *an MRS ρ (denoted $\sigma \overset{*}{\Rightarrow} \rho$) iff there exist MRSs σ', ρ' such that $\sigma \sqsubseteq \sigma'$, $\rho \sqsubseteq \rho'$ and $\sigma' \overset{*}{\rightarrow} \rho'$.*

Definition 2.9 (Language). *The **language** of a unification grammar G is $L(G) = \{w \in \Sigma^* \mid w = a_1 \cdots a_n$ and $\langle A^s \rangle \overset{*}{\Rightarrow} \langle A_1, \ldots, A_n \rangle \}$, where $A_i \in \mathcal{L}(a_i)$ for $1 \leq i \leq n$.*

Figure 2.2 lists an example unification grammar, G_{ww}, where $L(G_{ww}) = \{ww \mid w \in \{a,b\}^*\}$. The feature WORD stands for a list of items, HD is the head of the list followed by TL. A lexical item is generated once the WORD list consists of exactly one item (*ta* or *tb*). The reentrancy in the first rule (denoted by $\boxed{1}$) guarantees that both branching nodes are mapped by δ to the same node using the feature WORD. The second rule removes items from a non-empty WORD list; at each application a lexical item is generated.

$$A^s = \left[\text{WORD} : \begin{bmatrix}\text{HD} : s \\ \text{TL} : \textit{elist}\end{bmatrix}\right]$$

$$\mathcal{R} = \left\{ \begin{array}{l} \left[\text{WORD} : \begin{bmatrix}\text{HD} : s \\ \text{TL} : \textit{elist}\end{bmatrix}\right] \longrightarrow \left[\text{WORD} : \boxed{1}\right] \quad \left[\text{WORD} : \boxed{1}\right] \\ \left[\text{WORD} : \begin{bmatrix}\text{HD} : \boxed{1} \\ \text{TL} : \boxed{2}\end{bmatrix}\right] \longrightarrow \left[\text{WORD} : \boxed{2}\right] \quad \left[\text{WORD} : \begin{bmatrix}\text{HD} : \boxed{1} \\ \text{TL} : \textit{elist}\end{bmatrix}\right] \end{array} \right\}$$

$$\mathcal{L}(a) = \left\{\left[\text{WORD} : \begin{bmatrix}\text{HD} : \textit{ta} \\ \text{TL} : \textit{elist}\end{bmatrix}\right]\right\} \qquad \mathcal{L}(b) = \left\{\left[\text{WORD} : \begin{bmatrix}\text{HD} : \textit{tb} \\ \text{TL} : \textit{elist}\end{bmatrix}\right]\right\}$$

Figure 2.2: An example unification grammar, G_{ww}.

Definition 2.10 (Derivation trees). *Let $G = \langle \mathcal{R}, \mathcal{L}, A^s \rangle$ be a unification grammar. A tree is a derivation (or parse) tree admitted by G iff:*

- *The root of the tree is the start symbol A^s;*
- *The internal vertices are feature structures (over the same features and atoms as the grammar G);*
- *The leaves are terminals;*
- *If a vertex A has k descendants, $B_1, B_2, \ldots, B_k$, then $\langle A \rangle$ immediately derives $\langle B_1, \ldots, B_k \rangle$ with respect to some rule $\rho \in \mathcal{R}$.*
- *If a vertex A dominates a terminal b then A is unifiable with some $B \in \mathcal{L}(b)$.*

Figure 2.3 lists an example derivation tree for the string $baabaa$ generated by the unification grammar, G_{ww}, of figure 2.2: in the first derivation step, the initial symbol is unifiable with the first rule's head, then, each of the resulting feature structures are unifiable with the second rule's head until all lexical symbols are generated.

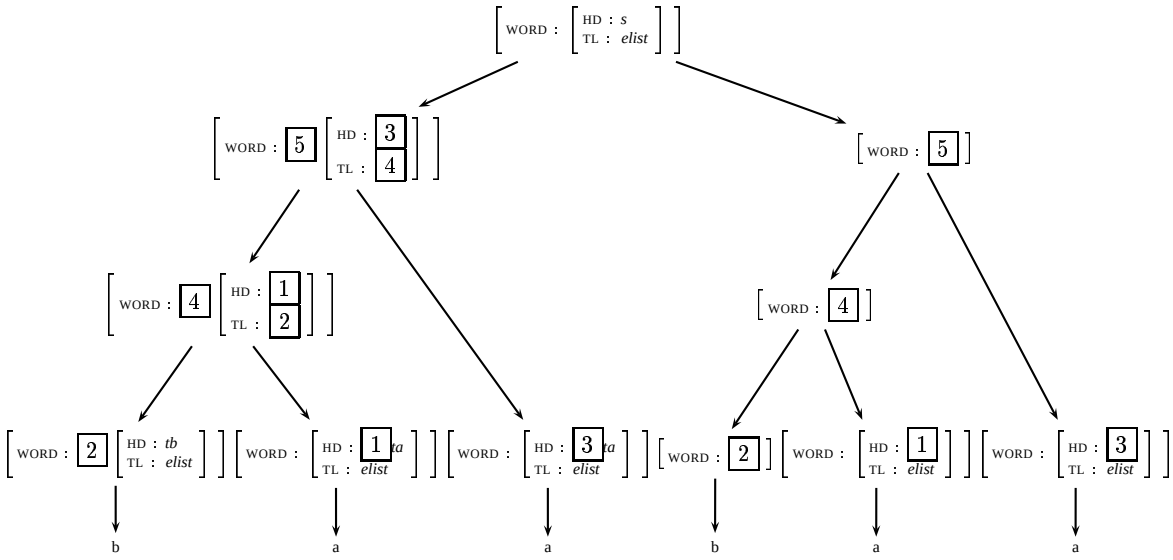


Figure 2.3: An example parse tree for $baabaa \in G_{ww}$

The expressive power of unification grammars Unification grammars are equivalent to Turing machines in their generative capacity. Determining whether a given string w is generated by a given grammar G is equivalent to deciding whether a Turing machine M halts on an empty input. Since the latter is undecidable, so is the recognition problem (Johnson, 1988).

2.3 Skeletal grammars

Skeletal grammars are a variant of unification grammars which assume an explicit *context-free backbone/skeleton*. These grammars can be viewed as an extension of context-free grammars, where every category is associated with an informative feature structure. The *context-free backbone* of a skeletal grammar is obtained by ignoring all feature structures of the grammar rules and considering only the categories.

An **extended category** is a pair $\langle A, c\rangle$ where A is a feature structure and $c \in$ CATS is a category.

Definition 2.11. *A **skeletal grammar** (over* FEATS, ATOMS *and* CATS*) is a tuple $G = \langle \mathcal{R}, \mathcal{L}, A^s\rangle$ where:*

- *$\mathcal{R}$ is a finite set of rules, each of which is an MRS of length $n \geq 1$ (with a designated first element, the head of the rule), and a sequence of length n of categories over the parameter* CATS *(The first category represents the head's category).*
- *$\mathcal{L}$ is a lexicon, which associates with every terminal a (over a fixed finite set Σ of terminal words) a finite set $\mathcal{L}(a)$ of extended categories $\langle A, C\rangle$, where A is a feature structure and $C \in$ CATS is a category.*

- $A^s = \langle A, S \rangle$ *is the start symbol (an extended initial category).*

A *skeletal form* is a pair $\langle \sigma, \vec{c} \rangle$, where σ is an MRS of length n and $\vec{c}$ is a sequence of n categories ($c_i \in$ CATS for $1 \leq i \leq n$).

Definition 2.12 (Derivation). *Let $\langle \sigma_A, \vec{c_A} \rangle$ and $\langle \sigma_B, \vec{c_B} \rangle$ be forms such that $\sigma_A = \langle A_1, \ldots, A_k \rangle$ and $\sigma_B = \langle B_1, \ldots, B_m \rangle$. $\langle \sigma_A, \vec{c_A} \rangle$ **immediately derives** $\langle \sigma_B, \vec{c_B} \rangle$ iff there exist a skeletal rule $\langle \rho', \vec{c_R} \rangle \in \mathcal{R}$ of length n, an MRS ρ, $\rho' \sqsubseteq \rho$, such that:*

- $m = k + n - 2$*;*
- *ρ's head is some element i of σ_A: $\rho^1 = \sigma_A^i$;*
- *ρ's body is a sub-structure of σ_B: $\rho^{2 \ldots n} = \sigma_B^{i \ldots i+n-2}$;*
- *The first $i-1$ elements of σ_A and σ_B are identical: $\sigma_A^{1 \ldots i-1} = \sigma_B^{1 \ldots i-1}$;*
- *The last $k-i$ elements of σ_A and σ_B are identical: $\sigma_A^{i+1 \ldots k} = \sigma_B^{m-(k-i+1) \ldots m}$;*
- *$\vec{c_B}$ is obtained by replacing the i-th element of $\vec{c_A}$ by the body of $\vec{c_R}$.*

The reflexive transitive closure of '$\rightarrow$' is denoted '$\overset{*}{\rightarrow}$'.

*A form $\langle \sigma_A, \vec{c_A} \rangle$ **derives** $\langle \sigma_B, \vec{c_B} \rangle$ (denoted $\langle \sigma_A, \vec{c_A} \rangle \overset{*}{\Rightarrow} \langle \sigma_B, \vec{c_B} \rangle$) iff there exist MRSs σ'_A, σ'_B such that $\sigma_A \sqsubseteq \sigma'_A$, $\sigma_B \sqsubseteq \sigma'_B$ and $\langle \sigma'_A, \vec{c_A} \rangle \overset{*}{\rightarrow} \langle \sigma'_B, \vec{c_B} \rangle$.*

Definition 2.13 (Language). *The **language** of a skeletal grammar G is $L(G) = \{ w \in \Sigma^* \mid w = a_1 \cdots a_n$ and $\langle A^s \rangle \overset{*}{\Rightarrow} \langle \langle A_1, \ldots, A_n \rangle, \langle C_1, \ldots, C_n \rangle \rangle \}$, where $\langle A_i, C_i \rangle \in \mathcal{L}(a_i)$ for $1 \leq i \leq n$.*

Figure 2.4 lists an example skeletal grammar, G_{abc}, where $L(G_{abc}) = \{a^n b^n c^n \mid n > 0\}$.

$$\text{CATS} = \{S, A, B, C\}$$

$$A^s = \langle [\text{LEN} : s], S \rangle$$

$$\mathcal{R} = \left\{ \begin{array}{llll} [\text{LEN} : s] \longrightarrow [\text{LEN} : \boxed{1}] & [\text{LEN} : \boxed{1}] & [\text{LEN} : \boxed{1}] & S \longrightarrow A\ B\ C \\ [\text{LEN} : [\text{LEN} : \boxed{1}]] \longrightarrow [\text{LEN} : \boxed{1}] & [\text{LEN} : elist] & & A \longrightarrow A\ A \\ [\text{LEN} : [\text{LEN} : \boxed{1}]] \longrightarrow [\text{LEN} : \boxed{1}] & [\text{LEN} : elist] & & B \longrightarrow B\ B \\ [\text{LEN} : [\text{LEN} : \boxed{1}]] \longrightarrow [\text{LEN} : \boxed{1}] & [\text{LEN} : elist] & & C \longrightarrow C\ C \end{array} \right\}$$

$$\mathcal{L}(a) = \{\langle [\text{LEN} : elist], A \rangle\} \quad \mathcal{L}(b) = \{\langle [\text{LEN} : elist], B \rangle\} \quad \mathcal{L}(c) = \{\langle [\text{LEN} : elist], C \rangle\}$$

Figure 2.4: An example skeletal grammar, G_{abc}.

Derivation trees for skeletal grammars are defined similarly to definition 2.10 of derivation trees for general unification grammars, except that every internal vertex consists of an extended

category instead of a feature structure. It is possible to obtain a context-free derivation tree from a skeletal derivation tree by ignoring the feature structures; the obtained context-free derivation tree is called a *constituent structure*.

Definition 2.14 (Derivation trees). *Let $G = \langle \mathcal{R}, \mathcal{L}, A^s \rangle$ be a skeletal grammar. A tree is a derivation tree admitted by G if:*

- *The root of the tree is the start symbol $A^s = \langle A, S \rangle$;*
- *The internal vertices are extended categories (over the same features, atoms and categories as the grammar G);*
- *The leaves are terminals;*
- *If a vertex $\langle A, c \rangle$ has k descendants, $\langle B_1, c_1 \rangle, \langle B_2, c_2 \rangle, \ldots, \langle B_k, c_k \rangle$, then $\langle \langle A \rangle, \langle c \rangle \rangle$ immediately derives $\langle \langle B_1, \ldots, B_k \rangle, \langle c_1, \ldots, c_k \rangle \rangle$ with respect to some rule $\langle \rho, \vec{c_R} \rangle \in \mathcal{R}$.*
- *If a vertex $\langle A, c \rangle$ dominates a terminal b then A is unifiable with some B, such that $\langle B, c \rangle \in \mathcal{L}(b)$.*

2.4 Comparison between general and skeletal unification grammars

An extended category $\langle A, c \rangle$ can be rewritten as a pure feature structure by *internalizing* the category into the feature structure, which is done by adding a new atom c to ATOMS and a new feature $\text{CAT} \notin \text{FEATS}$ to the set of features FEATS, such that for every feature structure A, $\delta(A, \text{CAT})$ is a sink and $\theta(\delta(A, \text{CAT})) = c$. For example, $\langle \begin{bmatrix} \text{WORD} : W \end{bmatrix}, c \rangle$ is internalized into $\begin{bmatrix} \text{CAT} : & c \\ \text{WORD} : W \end{bmatrix}$ Therefore, any skeletal unification grammar can be represented as a general unification grammar.

Every general unification grammar yields a trivial context-free backbone (which is retrieved by adding a meta-category, C, which is attached to all feature structures), but such a context-free backbone provides no information regarding the constituents within a sentence. General unification grammars do not necessarily assume an explicit (non-trivial) context-free backbone. A non-trivial context-free backbone can be extracted (if it exists) by mapping combinations of feature values to categories (for example, $f\left(\begin{bmatrix} \text{G} : a \\ \text{H} : b \end{bmatrix}\right) = C_1$, $f\left(\begin{bmatrix} \text{G} : b \\ \text{H} : b \end{bmatrix}\right) = C_2$), but there exist no algorithm for retrieving it.

A variety of unification-based grammar formalisms exists, some assume an explicit context-free backbone, for example, PATR-II (Shieber, 1986), Definite Clause Grammar, DCG (Pereira and Warren, 1980), Lexical Functional Grammar, LFG (Kaplan and Bresnan, 1982), others do not, for example, Generalized Phrase Structure Grammar, GPSG (Gazdar et al., 1985) and Head-Driven Phrase Structure Grammar, HPSG (Pollard and Sag, 1986).

2.5 Some grammar examples

In this section we give some grammar examples which will be used in the rest of the thesis. In order to simplify the examples and avoid the usage of complicated feature structures, the examples

use a straightforward encoding of lists as feature structures, where a list is represented by brackets, the list items are separated by a comma, an empty list is denoted by $\langle\ \rangle$ and $\langle head \mid tail \rangle$ represents a list whose first item is $head$ followed by $tail$. For example, (a transformation from a list feature structure to the brackets notation):

$$\left[\text{WORD} : \left[\begin{array}{l}\text{HD} : tb \\ \text{TL} : \left[\begin{array}{l}\text{HD} : tb \\ \text{TL} : \left[\begin{array}{l}\text{HD} : tb \\ \text{TL} : elist\end{array}\right]\end{array}\right]\end{array}\right]\right] \text{ is shortened to } \left[\text{WORD} : \langle tb\ ,\ tb\ ,\ tb \rangle\right]$$

Figure 2.5 lists an example unification grammar generating the language $\{b^+\}$. The feature CAT stands for the category, and WORD is a list of lexical symbols. The string b is the only terminal item in the lexicon, therefore every string generated by the grammar consists of b's only. The grammar is unambiguous; a string of N occurrences of b has exactly one parse tree. The second rule adds items to a list at each derivation step. The fourth rule removes items from the list. With each removal derivation step a b is generated until a list of one item is reached. A derivation tree for a string of N occurrences of b has a linear depth of $2N$. Figure 2.6 lists a derivation tree example.

$$A^s = \left[\begin{array}{l}\text{CAT} : s \\ \text{WORD} : \langle s \rangle\end{array}\right]$$

$$\mathcal{R} = \left\{\begin{array}{ll}
(1) & \left[\begin{array}{l}\text{CAT} : s \\ \text{WORD} : \langle s \rangle\end{array}\right] \longrightarrow \left[\begin{array}{l}\text{CAT} : p \\ \text{WORD} : \langle tb \rangle\end{array}\right] \\
(2) & \left[\begin{array}{l}\text{CAT} : p \\ \text{WORD} : \boxed{1}\end{array}\right] \longrightarrow \left[\begin{array}{l}\text{CAT} : p \\ \text{WORD} : \langle tb \mid \boxed{1} \rangle\end{array}\right] \\
(3) & \left[\begin{array}{l}\text{CAT} : p \\ \text{WORD} : \boxed{1}\end{array}\right] \longrightarrow \left[\begin{array}{l}\text{CAT} : q \\ \text{WORD} : \boxed{1}\end{array}\right] \\
(4) & \left[\begin{array}{l}\text{CAT} : q \\ \text{WORD} : \langle tb \mid \boxed{1} \rangle\end{array}\right] \longrightarrow \left[\begin{array}{l}\text{CAT} : q \\ \text{WORD} : \boxed{1}\end{array}\right] \quad \left[\begin{array}{l}\text{CAT} : q \\ \text{WORD} : \langle tb \rangle\end{array}\right]
\end{array}\right\}$$

$$\mathcal{L}(b) = \left\{\left[\begin{array}{l}\text{CAT} : q \\ \text{WORD} : \langle tb \rangle\end{array}\right]\right\}$$

Figure 2.5: A unification grammar, G_{FA}.

By the value of the feature CAT, the grammar rules must be applied according to their order. The first rule is applied at most once (in fact, exactly once, since the initial symbol's category is S), then since the current category is P, either the second or third rule may be applied. The second rule may be applied any number of times, adding an item to the list at each derivation step, but once the third rule has been applied, the second rule is no longer applicable since the current category is Q. The third rule is applied exactly once, and then the only applicable rule is the fourth rule. The fourth rule is applied repeatedly until a list of one item is reached, generating a lexical item at each derivation step. Therefore, the only possible derivation for the string b^l is by one application of the first rule, followed by $l-1$ consecutive applications of the second rule,

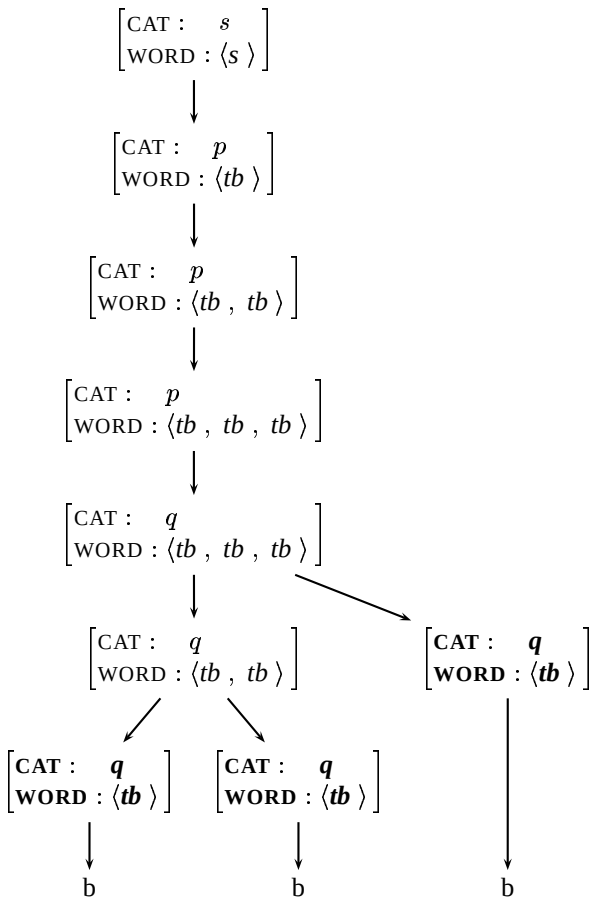


Figure 2.6: An example derivation tree for the grammar G_{FA} of figure 2.5 and the string *bbb*.

followed by one application of the third rule and then $l-1$ applications of the fourth rule. Thus, a string of l occurrences of b has just one parse tree.

Lemma 2.1. *There exists a derivation tree for the string b^l ($l > 0$), admitted by G_{FA}, of depth $2l$, in which the first rule is applied once, then the second rule is applied $l-1$ times resulting in a* WORD *list of l items, then an application of the third rule and then $l-1$ application of the fourth rule, each application of the fourth rule removes an item from the* WORD *list and a b is generated until a list of one item is reached.*

Proof. The proof is by induction on l the size of the derived string. For $l = 1$, figure 2.7 lists a derivation tree for the string b satisfying the conditions.

Assume that the induction hypothesis holds for all k, $1 \leq k < l$. For $k = l$ ($l > 1$): by the induction hypothesis there exists a derivation tree for the string b^{l-1} in which the second rule is applied $l-2$ times, resulting in a list of $l-1$ items. Applying the second rule once more results in a list of l items. By the induction hypothesis at each application of the fourth rule an item is removed from the list and a b is generated. Thus applying the fourth rule $l-2$ times results in $l-2$ b's and a list of two items. Since a b is mapped in the lexicon to the category Q ($[\text{CAT} : q]$) and a

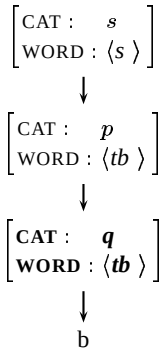


Figure 2.7: A derivation tree admitted by the grammar G_{FA} for the string b.

list of one item, one more application of the fourth rule would result in two more b's. Hence, in order to generate the string b^l, the first rule is applied once, the second rule is applied $l-1$ times, resulting in a list of l items, the third rule is a applied once and then the fourth rule is applied $l-1$ times. □

Corollary 2.2. *The grammar G_{FA} generates the language $L = \{b^l \mid l > 0\}$*

Proof. Since the string b is the only terminal item in the lexicon, and the grammar contains no ϵ-rules, any derived string consists of b's only. By lemma 2.1, there exists a derivation tree for the string b^l for any $l > 0$. □

Figures 2.8 lists an example unification grammar generating the language $\{b\}$. The feature CAT stands for the category, and WORD is a list of lexical symbols. The grammar rules must be applied according to their order as defined by the values of the feature CAT. The second rule adds items to the WORD list, the fourth rule removes items from the list, a b is generated once the list consists of one item. Therefore there exist infinitely many derivation trees, of arbitrary depths, for the string b, for any natural number of applications of the second rule. The derivation trees are of the form sketched in figure 2.9,

Lemma 2.3. *The grammar G_{inf} generates the language $L = \{b\}$.*

Proof. The grammar consists of unit-rules only, therefore it can only generate non-branching derivation trees whose frontier consists of one symbol. Since the string b is the only terminal item at the lexicon, and the grammar contains no ϵ-rules, any derived string consists of b only. Figure 2.10 lists a derivation tree for the string b, therefore $L(G_{inf}) = \{b\}$. □

Figure 2.11 lists an example unification grammar generating the language $\{b^+\}$. The string b is the only terminal item at the lexicon and there exist no ϵ-rules, therefore every string generated by the grammar consists of b's only. A string of N occurrences of b has exactly one parse tree. The depth of the derivation tree is 2^N. The feature CAT stands for the category, the feature depthCount is a list that represents the current depth of the derivation tree; the number of derivation steps from the root. In each derivation step an item is added to the depthCount list, but no items may ever be

$$A^s = \begin{bmatrix} \text{CAT}: & s \\ \text{WORD}: & \langle s \rangle \end{bmatrix}$$

$$\mathcal{R} = \left\{ \begin{array}{ll} (1) & \begin{bmatrix} \text{CAT}: & s \\ \text{WORD}: & \langle s \rangle \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \\ (2) & \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \boxed{1} \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \mid \boxed{1} \rangle \end{bmatrix} \\ (3) & \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \boxed{1} \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \boxed{1} \end{bmatrix} \\ (4) & \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \langle tb \mid \boxed{1} \rangle \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \boxed{1} \end{bmatrix} \end{array} \right\}$$

$$\mathcal{L}(b) = \left\{ \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \right\}$$

Figure 2.8: A unification grammar, G_{inf}.

removed from it. The feature innerCount is a list that represents the number of derivation steps before generating the next b symbol. Every application of the second rule doubles the depth of the innerCount list (with respect to its length after the previous application of the rule). Thus the number of derivation steps for generating each b is always twice the number of steps for generating its predecessor.

The grammar derivation steps are as follows:

1. The first derivation step (and the only possible one) is by the first rule, adding one item to depthCount list, leaving the innerCount list empty.

2. Since the innerCount list is empty, either the second or fourth rules may be applied: by applying the second rule, the depthCount list is assigned into the innerCount list, an item is added to depthCount list and a b is generated. Once the second rule has been applied, the third rule is the only applicable rule, since innerCount is non-empty. By applying the fourth rule, a b is generated and the parsing is terminated.

3. Then, the third rule is applied (assuming that the second rule has been previously applied), removing items from the innerCount list, until an empty list is reached, then go back to 2.

4. By applying the fourth rule the last b is generated.

Figure 2.12 lists a derivation tree example.

Lemma 2.4. *There exists a derivation tree for the string b^l ($l > 0$) of depth 2^l.*

Proof. The proof is by induction on l, the number of b lexical symbols. For $l = 1$, figure 2.13 lists an example derivation tree for the string b satisfying the conditions.

Assume that the induction hypothesis holds for all k, $1 \leq k < l$. For $k = l$: by the induction hypothesis, a string of $l - 1$ occurrences of b has a derivation tree of depth 2^{l-1}. By the grammar

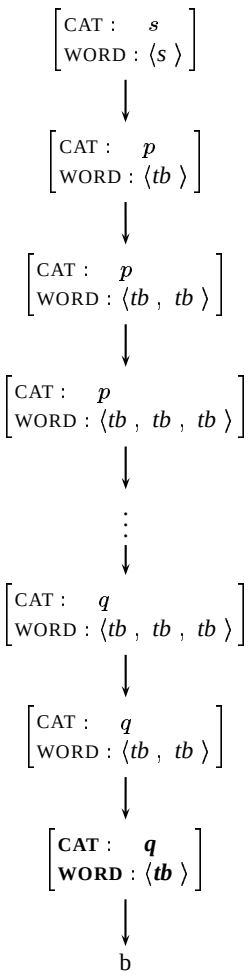


Figure 2.9: The derivation tree form of the grammar G_{inf} of figure 2.8.

rules, a b is generated only when the innerCount list is empty. Thus the only possibilities for generating a b is by applying either the second or fourth (terminating) rules. Therefore, it can be deduced that the $(l-1)^{th}$ b, in a derivation tree for the string b^l, was generated by the second rule.

By the induction hypothesis, the depth of the derivation tree for generating $l-1$ symbols is 2^{l-1}, thus before the immediate derivation of the $(l-1)^{th}$ symbol, the depthCount list contained $2^{l-1}-1$ items (since depthCount list represents a counter of the derivation's depth). While generating the $(l-1)^{th}$ symbol by an application of the second rule, the depthCount list was assigned into the innerCount list, therefore the innerCount list contains $2^{l-1}-1$ items.

After an application of the second rule (generating the $(l-1)^{th}$ symbol), since the innerCount list is non-empty, the only applicable rule is the third rule. Thus, $|innerCount|$ $(= 2^{l-1}-1)$ applications of the third rule must be applied until the innerCount list is empty. Then the fourth rule can be applied once more in order to generate the l^{th} terminating symbol.

There exist 2^{l-1} derivation steps for generating the first $l-1$ symbols, and 2^{l-1} derivation

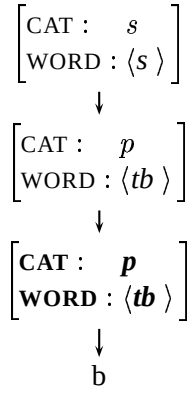


Figure 2.10: An example derivation tree for the grammar G_{inf} of figure 2.8 and the string b.

$$A^s = \begin{bmatrix} \text{CAT} : & s \\ depthCount : & \langle\ \rangle \\ innerCount : & \langle\ \rangle \end{bmatrix}$$

$$\mathcal{R} = \left\{ \begin{array}{ll} (1) & \begin{bmatrix} \text{CAT} : & s \\ depthCount : & \langle\ \rangle \\ innerCount : & \langle\ \rangle \end{bmatrix} \rightarrow \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb\ \rangle \\ innerCount : & \langle\ \rangle \end{bmatrix} \\ (2) & \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \boxed{1} \\ innerCount : & \langle\ \rangle \end{bmatrix} \rightarrow \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb \mid \boxed{1}\ \rangle \\ innerCount : & \boxed{1} \end{bmatrix} \quad \begin{bmatrix} \text{CAT} : l \\ lex : tb \end{bmatrix} \\ (3) & \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \boxed{1} \\ innerCount : & \langle tb \mid \boxed{2}\ \rangle \end{bmatrix} \rightarrow \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb \mid \boxed{1}\ \rangle \\ innerCount : & \boxed{2} \end{bmatrix} \\ (4) & \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb \mid \boxed{1}\ \rangle \\ innerCount : & \langle\ \rangle \end{bmatrix} \rightarrow \begin{bmatrix} \text{CAT} : l \\ lex : tb \end{bmatrix} \end{array} \right\}$$

$$\mathcal{L}(b) = \left\{ \begin{bmatrix} \text{CAT} : l \\ lex : tb \end{bmatrix} \right\}$$

Figure 2.11: A unification grammar, G_{DB}.

steps for generating the l^{th} symbol. Therefore, there exist a derivation tree for the string b^l of depth 2^l. □

Corollary 2.5. *The grammar G_{DB} generates the language $L = \{b^l \mid l > 0\}$*

The only possible derivation tree for a string of l occurrences of b consists of exactly $l - 1$ applications of the second rule, each followed by $|innerCount|$ applications of the third rule (until innerCount list is empty) and then one application of the fourth (terminating) rule. Thus a string of l occurrences of b has just one parse tree.

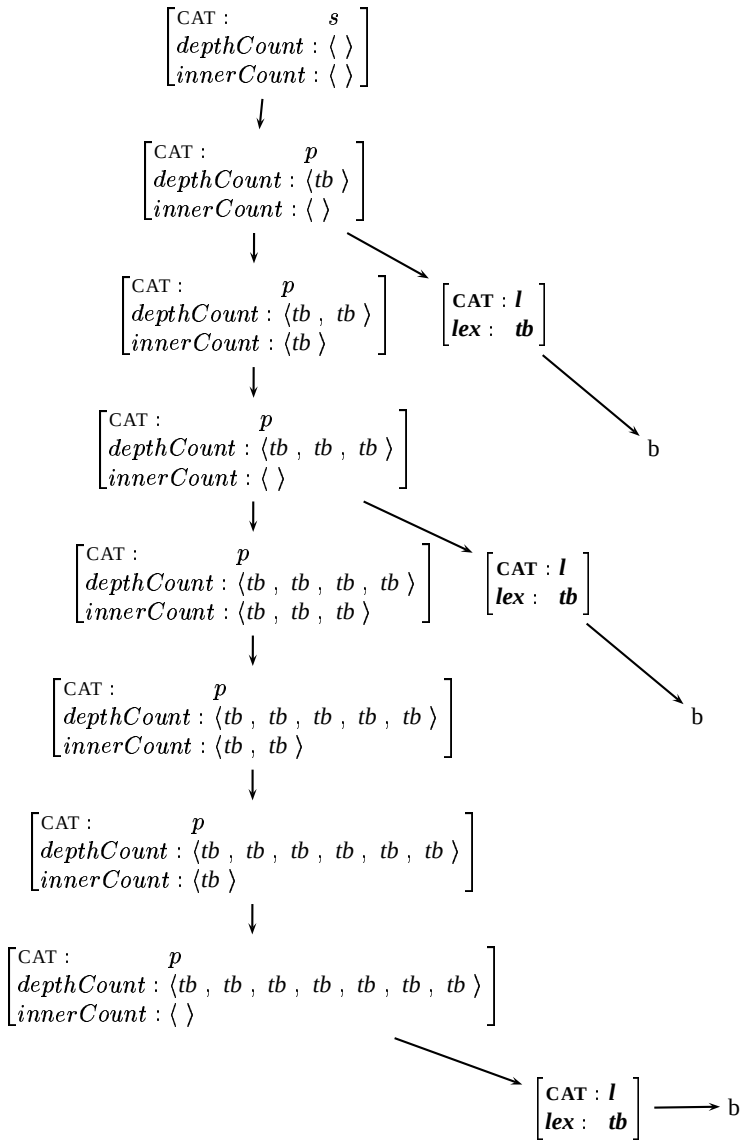


Figure 2.12: An example derivation tree for the grammar G_{DB} of figure 2.11 and the string *bbb*. The tree's depth is 2^3.

Figure 2.14 lists a unification grammar generating the language $\{b^{2^l} \mid l > 0\}$. It is a variation of G_{DB} with an addition that in every derivation step a lexical item is generated. Thus the grammar generates each string of 2^N symbols in 2^N derivation steps.

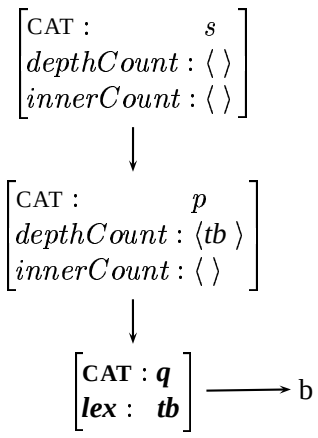


Figure 2.13: A derivation tree admitted by the grammar G_{DB} for the string b.

$$A^s = \begin{bmatrix} \text{CAT} : & s \\ depthCount : & \langle\ \rangle \\ innerCount : & \langle\ \rangle \end{bmatrix}$$

$$\mathcal{R} = \left\{ \begin{array}{l} \begin{bmatrix} \text{CAT} : & s \\ depthCount : & \langle\ \rangle \\ innerCount : & \langle\ \rangle \end{bmatrix} \rightarrow \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb\ \rangle \\ innerCount : & \langle\ \rangle \end{bmatrix} \quad \begin{bmatrix} \text{CAT} : l \\ lex : \ tb \end{bmatrix} \\ \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \boxed{1} \\ innerCount : & \langle\ \rangle \end{bmatrix} \rightarrow \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb \mid \boxed{1}\ \rangle \\ innerCount : & \boxed{1} \end{bmatrix} \quad \begin{bmatrix} \text{CAT} : l \\ lex : \ tb \end{bmatrix} \\ \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \boxed{1} \\ innerCount : & \langle tb \mid \boxed{2}\ \rangle \end{bmatrix} \rightarrow \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb \mid \boxed{1}\ \rangle \\ innerCount : & \boxed{2} \end{bmatrix} \quad \begin{bmatrix} \text{CAT} : l \\ lex : \ tb \end{bmatrix} \\ \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb \mid \boxed{1}\ \rangle \\ innerCount : & \langle\ \rangle \end{bmatrix} \rightarrow \begin{bmatrix} \text{CAT} : l \\ lex : \ tb \end{bmatrix} \end{array} \right\}$$

$$\mathcal{L}(b) = \left\{ \begin{bmatrix} \text{CAT} : l \\ lex : \ tb \end{bmatrix} \right\}$$

Figure 2.14: A unification grammar, $G_{b^{2^n}}$.

Chapter 3

Off-line parsability constraints

It is well known that unification based grammar formalisms are Turing equivalent in their generative capacity (Pereira and Warren, 1983; Johnson, 1988, 87-93); determining whether a given string w is generated by a given grammar G is equivalent to deciding whether a Turing machine M halts on an empty input ϵ, which is known to be undecidable. Thus, the recognition problem is undecidable in the general case. Therefore, a constraint on grammars was suggested which guarantees decidability of the recognition problem for a subset of all grammars; this constraint is generally known as the off-line parsability constraint (OLP), and several variants of OLP definitions are known.

In this chapter we present several variants of OLP constraints suggested in the literature. Some of the constraints (Pereira and Warren, 1983; Kaplan and Bresnan, 1982; Johnson, 1988; Kuhn, 1999) apply only to skeletal grammars since they explicitly refer to categories, which are undetermined for general unification grammars. Others (Haas, 1989; Shieber, 1992; Torenvliet and Trautwein, 1995; Francez and Wintner, 1999) are applicable to both skeletal and general unification grammars. Some of the OLP constraints refer to OLP grammars, while others impose a restriction on allowable derivation trees (OLP derivations), but provide no explicit definition of OLP grammars. In this research we are concerned with OLP grammars, therefore, in section 3.1, we extend these definitions from derivations to grammars.

3.1 Off-line parsability variants

We begin the discussion with the OLP constraints for skeletal grammars. One of the first definitions was suggested by Pereira and Warren (1983). Their constraint was designed for DCGs (where the predicate names constitute an explicit context-free backbone) for guaranteeing termination of general proof procedures for definite clause sets. Rephrased in terms of skeletal grammars, the definition is as follows:

Definition 3.1 (Pereira and Warren's OLP constraint for skeletal grammars (OLP_{PW})). *A skeletal grammar is off-line parsable iff its context-free skeleton is not infinitely ambiguous.*

The *context-free skeleton* of a skeletal grammar is obtained by ignoring all feature structures of the grammar rules and considering only the categories. In the next section we prove that the depth of every derivation tree generated by a grammar whose context-free skeleton is finitely ambiguous

is bounded by the number of syntactic categories times the size of its yield, and that makes the recognition problem decidable.

The next OLP definition is based on Kaplan and Bresnan (1982). Kaplan and Bresnan suggested a linguistically motivated OLP constraint which refers to valid derivations for the Lexical-Functional Grammar formalism (LFG), a skeletal grammar formalism. Unlike the previous OLP definition, in LFG the notion of an OLP **grammar** does not exists. Instead, LFG defines OLP **derivations**: for a given string w, a derivation tree is not OLP if it has two nodes, u and v, where u dominates v and u and v span exactly the same substring of w. Of course, there can be other derivations of w which are OLP.

In LFG, the definition of $L(G)$ is not the standard one. Rather, w is in $L(G)$ if there exists an OLP derivation tree for w; the structures associated with w by G are the structures induced by the OLP derivations only. Other possible (non-OLP) derivations are simply ignored.

LFG introduces two kinds of ϵ's, controlled and optionality ϵ's, which are used in descriptions of natural languages. General unification grammars are not necessarily designed for natural languages and thus the distinction between the ϵ kinds does not necessarily exist. Hence, we use a variant of Kaplan and Bresnan's constraint, suggested by Johnson (1988, 95-97), eliminating all ϵ's of any kind.

Definition 3.2 (Johnson's OLP constraint (OLP_{JO})). *A constituent structure satisfies Johnson's off-line parsability constraint iff:*

- *It does not include a non-branching dominance chain in which the same category appears twice.*
- *The empty string ϵ does not appear as a lexical form annotation of any (terminal) node.*

The first condition rules out any unbounded unary branching structures, whereas the second condition rules out any applications of ϵ-rules. In the next section we prove that the constraint bounds the depth of any OLP derivation tree by a linear function of the size of its yield, thus ensuring decidability of the recognition problem.

Johnson's constraint, as well as the next OLP variant discussed below, are based on Kaplan and Bresnan's OLP for LFG and thus they impose a restriction on allowable c-structures, rather than on the grammar itself. There exist no explicit definition of an OLP grammar. Such a definition can be understood in (at least) two manners:

Definition 3.3 (OLP_{Δ} grammar).

1. *A grammar G is off-line parsable iff for every $w \in L(G)$ **every** derivation tree for w satisfies OLP_{Δ}.*
2. *A grammar G is off-line parsable iff for every $w \in L(G)$ **there exists** a derivation tree which satisfies OLP_{Δ}.*

Note that the OLP grammar definitions refer to the standard definition of $L(G)$, and OLP_{Δ} refers to some OLP derivations' definition (in this case Δ substitutes for JO). The first definition

is very strict, it allows grammars which can generate only OLP derivation trees. The second definitions is more liberal, it allows non-OLP derivation trees as long as there exists at least one OLP derivation tree for every word of the grammar's language. We refer to grammars that satisfy the first and second definitions of OLP_{JO} grammars by OLP_{JO_1} and OLP_{JO_2} respectively.

Johnson's definition of OLP constituent structures applies only to skeletal grammars. For general unification grammars the term **category** is not well defined since there may be infinitely many feature structures (there may not exist a mapping of the feature structures to a finite set of categories). In order to redefine the constraint for general unification grammars, the notion of 'category' should be defined first in terms of feature structures.

The next constraint is also based on Kaplan and Bresnan's constraint and is also dealing only with OLP derivations; OLP grammar definitions are according to definition 3.3. The constraint uses the notion of 'categories' and thus is applicable only to skeletal grammars.

X-bar theory grammars (Chomsky, 1975) have a strong linguistic justification in describing natural languages. Unfortunately neither Kaplan and Bresnan's nor Johnson's constraints allow such derivations, since they do not allow derivation trees in which the same category appears twice in a non-branching dominance chain. Kuhn (1999) refers to the problem from a linguist's point of view. The purpose of his constraint is to expand the class of derivation trees which satisfy Kaplan and Bresnan's constraint in order to allow X-bar derivations. As Kuhn (1999) does not explicitly refer to ϵ-rules, we assume that ϵ does not represent a lexical item, as in Johnson's constraint.

Definition 3.4 (Kuhn's OLP constraint). *A c-structures derivation is valid iff no category appears twice in a non-branching dominance chain with the same* ***f-annotation****.*

The c-structure in LFG represents the external structure of a sentence. It consists of a context-free derivation tree with an additional functional information about the constituents within a sentence. The functional information is represented by two meta-variables $\uparrow$ (up arrow) and $\downarrow$ (down arrow):

- Up arrow, $\uparrow$, represents the feature structure attached to the mother's node.
- Down arrow, $\downarrow$, represents the feature structure attached to the daughter node (the node itself).
- $(\uparrow \text{FEAT}) = \downarrow$, indicates that the value of the feature FEAT at the feature structure attached to the mother's node is the feature structure attached to the daughter node; $\delta(M, \text{FEAT}) = D$, where M, D represent the feature structures attached to the mother and daughter nodes, respectively.
- $\uparrow$=$\downarrow$, states that the mother and daughter nodes share the same feature structure.

The c-structures are then mapped into *f-structures*. The f-structure is an acyclic feature structure which represents the internal structure of a sentence, it includes a representation of the higher syntactic and functional information of a sentence. A detailed description of LFG's c-structures, f-structures and the transformation between them is given in Kaplan and Bresnan (1982).

Figure 3.1 lists an example of an X-bar derivation tree (c-structure) taken from Kuhn (1999) and its corresponding f-structure. The derivation tree contains a non-branching dominance chain in which the category VP appears twice. Therefore it is an invalid OLP derivation (by both Kaplan and Bresnan's and Johnson's constraints). Since the functional constraint attached to the daughter node is $(\uparrow\ xcomp) = \downarrow$, the mother and daughter nodes do not share the same f-annotation. Therefore, the derivation satisfies Kuhn's OLP constraint.

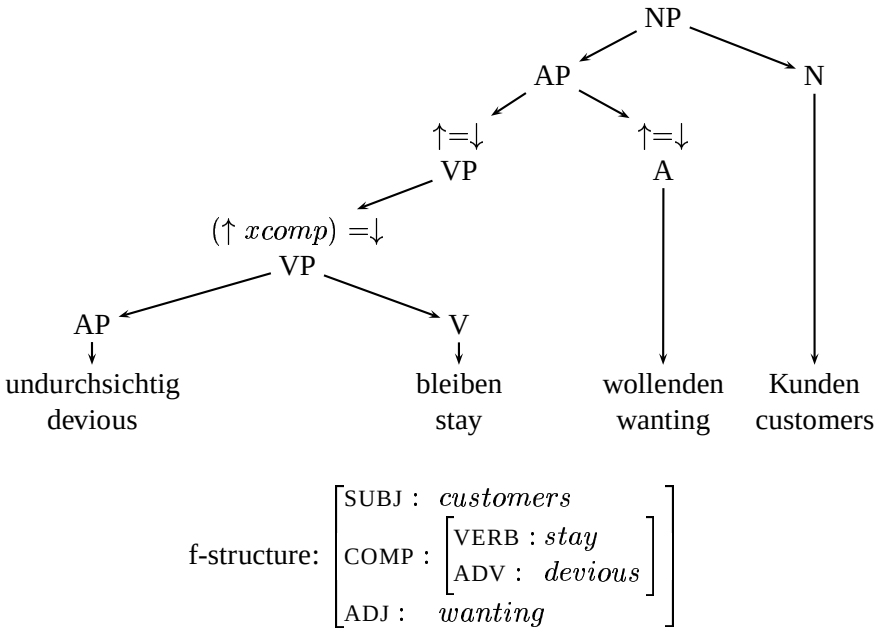


Figure 3.1: An example X-bar theory c-structure and f-structure of a German sentence.

Kuhn's constraint is defined in syntactic terms that are explicitly intended for the architecture of LFG grammars. Since we cannot translate them into our terminology, we exclude Kuhn's OLP constraint from further analysis.

The following definitions are applicable to both skeletal and general unification grammars. The first constraint was suggested by Haas (1989). Based on the observation that not every natural unification grammar has an obvious context-free backbone, Haas suggested a constraint for guaranteeing the solvability of the parsing problem which is applicable to all unification grammar formalisms.

Haas' definition of a derivation tree is slightly different from the one given in chapter 2 for unification grammars' derivation trees (definition 2.10). He allows derivation trees with nonterminals at their leaves, therefore a tree may represent a partial derivation.

Definition 3.5 (Haas' Depth-boundedness (DB)). *A unification grammar is depth-bounded iff for every $L > 0$ there is a $D > 0$ such that **every** parse tree for a sentential form of L symbols has depth less than D.*

Haas' definition of a depth-bounded grammar requires the existence of a function f such that

every derivation tree's depth for a sentential form of l symbols is bounded by $f(l)$. Since he allows partial derivation trees, a sentential form of length l may contain some non-terminal symbols.

According to Haas (1989), "a depth-bounded grammar cannot build an unbounded amount of tree structure from a bounded number of symbols", therefore, for every sentential form of length l there exist a finite number of partial derivation trees.

Haas shows an algorithm that should verify whether a given grammar is depth-bounded. But the algorithm's termination is not guaranteed; if depth-boundedness does not hold the algorithm will enter an infinite loop. Haas states explicitly that depth-boundedness is undecidable, but provides no proof of it.

We now refer back to OLP_{PW}. Recall that it applies only to skeletal grammars, as general unification grammars do not necessarily have an explicit context-free skeleton. The natural extension of Pereira and Warren's definition in order to apply to all unification grammar formalisms is finite ambiguity for unification grammars:

Definition 3.6 (Finite ambiguity for unification grammars (FA)). *A unification grammar G is OLP iff for every string w there exist only a finite number of derivation trees.*

Another OLP definition is suggested by Shieber (1992, 79–82) and is defined in terms of logical constraint based grammar formalisms. His constraint is defined in logical terms, such as models and operations on models. Rephrased in our terms, his definition is as follows:

Definition 3.7 (Shieber's OLP (OLP_S)). *A grammar G is off-line parsable iff there exists a finite-ranged function F on feature structures such that $F(A) \sqsubseteq A$ for all A and there are no derivation trees admitted by G in which a node A dominates a node B, both are roots of sub-trees with an identical yield and $F(A) = F(B)$.*

The constraint is intended to bound the depth of every derivation tree by the range of F times the size of its yield.

Johnson's OLP constraint is too restrictive, since it excludes all repetitive unary branching chains and ϵ- rules, furthermore; it is applicable only to skeletal grammars. Therefore, Torenvliet and Trautwein (1995) suggested a more liberal constraint, which is applicable to all unification grammar formalisms.

Definition 3.8 (Honest parsability constraint (HP)). *A grammar G satisfies the Honest Parsability Constraint (HPC) iff there exists a polynomial p s.t. for each $w \in L(G)$ there **exists** a derivation with at most $p(|w|)$ steps.*

The definition guarantees that for every string of the grammar's language there exist at least one polynomial depth (in the size of the derived string) derivation tree. Furthermore, the definition allows X-bar theory derivation trees, since a category may appear twice in a non-branching dominance chain as long as the depth of the tree is bounded by a polynomial function of its yield.

3.2 Off-line parsability analysis

In the following section we analyze the above given OLP definitions, define their properties and determine whether each of them guarantees decidability of the recognition problem.

Pereira and Warren's definition guarantees that the depth of every derivation tree admitted by an OLP_{PW} grammar is bounded by the number of syntactic categories in the grammar times the yield of the tree.

Lemma 3.1. *Let G be an OLP_{PW} grammar. Let G' be the context-free backbone of G (by definition G' is finitely ambiguous). G' cannot admit a derivation tree in which two sub-trees have the same root category and the same yield.*

Proof. Assume towards a contradiction that G', a finitely ambiguous context-free grammar, can generate such a derivation tree. Therefore, a sub-tree rooted by some category A ($\in$ CATS) may be generated repeatedly many times (by applying the same grammar rules), as shown in figure 3.2, resulting in infinite ambiguity. □

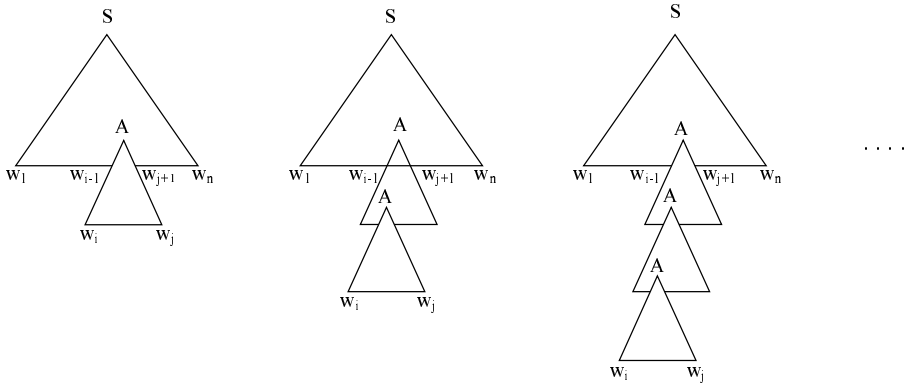


Figure 3.2: An example of derivation trees in which A is the root category of a sub-tree which dominates another sub-tree with root A and the same yield.

A skeletal derivation tree is a context-free derivation tree extended by feature structures. Therefore, G itself cannot admit a derivation in which a sub-tree dominates another sub-tree and both have the same root category and the same yield (otherwise by ignoring the feature structures, leaving just the categories, the context-free backbone would be infinitely ambiguous).

Corollary 3.2. *The depth of any OLP_{PW} derivation tree for a string of l symbols is bounded by a linear function of l.*

Proof. Let G be an OLP_{PW} grammar. By lemma 3.1, the maximum depth of a sub-derivation spanning a given yield is bounded by $|\text{CATS}|$, thus in every derivation tree after at most $|\text{CATS}|$ derivation steps, in which all nodes dominates the same yield, there must be either a terminating

node or an application of a rule expanding the yield. Therefore, in order to generate a string of l symbols, the depth of every derivation tree is at most $|\text{CATS}| \times l$. □

Since the depth of every derivation tree admitted by an OLP_{PW} grammar is bounded by a linear function of the size of its yield, it is possible to enumerate the derivation trees that have a given string as their yield, therefore parsing termination and decidability of the recognition problem are guaranteed.

Johnson's definition of OLP derivation trees guarantees that if there are no unary branching chains nor ϵ-rules, the number of rule applications in a derivation is linear in the length of the derived string and the size of the structure corresponding to the partial productions is polynomial. Therefore, decidability of the recognition problem is guaranteed.

Lemma 3.3. *The depth of any OLP_{JO} derivation tree for a string of l symbols is bounded by a linear function of l.*

Proof. Let d be an OLP_{JO} derivation tree for a string of l symbols:

- Due to the first condition of OLP_{JO}, d cannot contain a non-branching dominance chain in which the same category appears twice, therefore the size of any non-branching dominance chain is bounded by the number of different syntactic categories $|\text{CATS}|$.
- Due to the second condition of OLP_{JO}, an OLP_{JO} derivation tree's yield may not contain any ϵ's, therefore for a string of l symbols, the yield of any derivation tree is exactly of length l.

Therefore, the depth of any sub-derivation tree spanning a given yield is bounded by CATS. Hence, in order to generate a string of l symbols, the depth of every derivation tree is at most $|\text{CATS}| \times l$. □

Corollary 3.4. *For a given string there exist a finite number of OLP_{JO} derivation trees.*

Proof. By lemma 3.3 the depth of any derivation tree for a given string is bounded by a linear function of the length of the string. There exists a finite set of categories and a finite set of rules, therefore, there exist a finite number of all possible combinations of categories up to some linear depth. □

It is possible to enumerate the finite number of derivation trees that have a given string as their yield, therefore parsing termination and decidability of the recognition problem are guaranteed for OLP_{JO_1} grammars.

Corollary 3.5. *If G is OLP_{JO_2} then for every string w there exists a derivation tree whose depth is at most $|w| \times |\text{CATS}|$ (where $|\text{CATS}|$ is the number of syntactic categories). Therefore, the recognition problem is decidable.*

Proof. By lemma 3.3 the depth of any OLP_{JO} derivation tree for a given string of length l is bounded by a $l \times |\text{CATS}|$. By definition of OLP_{JO_2} grammars, for every $w \in L(G)$ there exists an OLP_{JO} derivation tree, thus for every string w there exists a derivation tree whose depth is at most $|w| \times |\text{CATS}|$. □

Haas's definition of depth-boundedness guarantees that there exist a function bounding the depth of every derivation tree admitted by G by the size of its yield. Suppose that such a function f was given, then for every string w of length l it was enough to check the finite set of derivation trees up to depth $f(w)$.

Finite ambiguity may play a role as the natural extension of the class of OLP_{PW} grammars to general unification grammars, but it does not necessarily guarantee decidability of the recognition problem; the fact that a grammar induces a finite number of derivation trees on every string provides us with no upper bound of the size of each derivation tree nor any information about the actual number of derivation trees that the grammar induces on every string. Therefore, for every string w and a given grammar G, it is possible to verify whether $w \in L(G)$ by generating a derivation tree for w, but since there exists no explicit number representing the finite number of derivation trees admitted by G for every string w, it is impossible to tell when a parsing algorithm may terminate while generating all derivation trees that G induces on w.

Shieber's OLP A grammar is OLP_S iff there exists a finite-ranged function mapping each two nodes dominating the same yield on every derivation tree to a different image, thus imposing a restriction on the depth of every derivation tree admitted by an OLP_S grammar limiting the number of derivation trees that have a given string as their yield to be finite.

Lemma 3.6. *The depth of any OLP_S derivation for a string of length l is at most $|range(F)| \times l$.*

Proof. Let G be an OLP_S grammar, therefore there exists a finite-ranged function F satisfying the OLP_S conditions. In each derivation tree, the number of derivation steps for generating each lexical item is bounded by $|range(F)|$ (otherwise at least two nodes are mapped to the same feature structure). Thus in order to generate a string of l symbols, the depth of every derivation tree is at most $|range(F)| \times l$. □

Torenvliet and Trautwein's HP grammars permit derivations in which there exists a non-branching dominance chain where the same category appears more than once, as long as the depth of the tree is bounded polynomially by the size of the derived string. Furthermore, an HP grammar allows the X-bar theory derivation tree of figure 3.1 and other X-bar derivation trees presented by Kuhn (1999).

HPC guarantees that there exist a polynomial function p such that for every $w \in L(G)$ there exists at least one derivation tree whose depth is bounded by $p(|w|)$. But the fact that there exists a polynomial function p provides no explicit information about p. Therefore, for every string w, it is possible to verify whether $w \in L(G)$ by generating a derivation tree for w, but since there is no explicit bound on the depth of the derivation tree, it is impossible to tell when a parsing algorithm may terminate while generating a derivation tree for w. Therefore it does not guarantee decidability of the recognition problem nor parsing termination.

Chapter 4

OLP definitions correlation

In the following section we make a comparison of the different OLP definitions discussed in the previous chapter using the grammar examples and lemmas of chapter 2. Such relationships were not investigated in the past. We first define for each of the given grammar examples their OLP properties. We then make comparative analysis of the OLP variants in terms of OLP grammars and define a hierarchy among them.

4.1 The grammar examples and OLP

Below we prove some lemmas regarding the given grammar examples of chapter 2, and their OLP properties. While the examples are all general unification grammars, they all have a context-free backbone (through the feature CAT), and thus we view them also as skeletal grammars when investigating the definitions which are applicable only to skeletal grammars.

The grammar G_{FA} of figure 2.5 (page 14).

Lemma 4.1. *G_{FA} is not OLP_{PW}.*

Proof. Figure 4.1 lists the extracted context-free backbone of G_{FA}. The context-free backbone contains the rule $P \longrightarrow P$, therefore, it can generate a unary branching chain of P's which immediately leads to infinite ambiguity. Therefore, G_{FA} is not an OLP_{PW} grammar. □

$$\mathcal{R} = \left\{ \begin{array}{l} S \longrightarrow P \\ P \longrightarrow P \\ P \longrightarrow Q \\ Q \longrightarrow Q\,Q \end{array} \right\}$$

Figure 4.1: The context-free backbone of the grammar G_{FA} of figure 2.5.

Lemma 4.2. *G_{FA} is neither OLP_{JO_1} nor OLP_{JO_2}.*

Proof. By lemma 2.1, in order to generate the string b^l for $l > 1$, exactly $l - 1$ applications of the second rule must be applied thus resulting in a non-branching dominance chain in which the category P appears more than once. Therefore not every $w \in L(G_{FA})$ has an OLP_{JO} derivation tree. □

Lemma 4.3. *G_{FA} is HP and FA.*

Proof. Since the grammar rules must be applied according to their order, a string of l occurrences of b has just one parse tree and its depth is $2l$. Therefore, for every string w there exists a finite number of derivation trees and there exists a polynomial depth (in $|w|$) derivation tree for every word of the grammar's language. Therefore the grammar is both HP and FA. □

Lemma 4.4. *G_{FA} is not DB.*

Proof. Haas' definition of a derivation tree allows derivations with non-terminals at their leaves. The grammar's second rule can be applied repeatedly many times, adding items to the list, generating arbitrarily deep derivation trees whose frontier has only one symbol. Therefore G_{FA} is not a DB grammar. □

Lemma 4.5. *G_{FA} is not OLP_S.*

Proof. In order to generate the string b^n, exactly $n - 1$ applications of the second rule must be applied, and then one application of the third rule, thus resulting in a non-branching dominance chain of length n. Suppose there exists a finite-ranged function, whose range is of size n, mapping each two nodes spanning the same yield to a different image (e.g. mapping each feature structure to itself). In order to generate the string b^{n+1} the function must map at least 2 nodes to the same feature structure, therefore there exists no finite-ranged function satisfying the constraint and the grammar is not an OLP_S grammar. □

The grammar G_{inf} of figure 2.8 (page 17).

Lemma 4.6. *G_{inf} is OLP_{JO_2}, but it is neither an OLP_{PW} nor an $OLPJO_1$ grammar.*

Proof. The grammar's language is $\{b\}$, figure 4.2 lists an example valid OLP_{JO} derivation for the string b, thus G_{inf} is OLP_{JO_2}. Figure 4.3 lists the extracted context-free backbone of G_{inf}. The context-free backbone contains the rule $P \longrightarrow P$, therefore, it can generate a unary branching chain of P's which immediately leads to infinite ambiguity. Furthermore, since G_{inf} may generate infinitely many derivation trees for the string b for any natural number of applications of the second rule, it may generate non-OLP_{JO} derivation trees. Therefore, G_{inf} is neither OLP_{PW} nor OLP_{JO_1}. □

Lemma 4.7. *G_{inf} is HP.*

Proof. The grammar's language is $\{b\}$, figure 4.2 lists a derivation tree for b of depth 3 generated by G_{inf}, therefore there exists a polynomial depth derivation tree for every $w \in L(G_{inf})$, thus the grammar is honest parsable. □

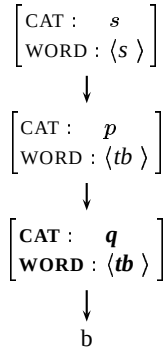


Figure 4.2: An OLP_{JO} derivation tree for the grammar G_{inf} of figure 2.8 and the string b.

$$\mathcal{R} = \left\{ \begin{array}{l} S \longrightarrow P \\ P \longrightarrow P \\ P \longrightarrow Q \\ Q \longrightarrow Q \end{array} \right\}$$

Figure 4.3: The context-free backbone of the grammar G_{inf} of figure 2.8.

Lemma 4.8. *G_{inf} is not FA.*

Proof. The grammar generates infinitely many derivation trees for the string b for any natural number of applications of the second rule, thus the grammar is infinitely ambiguous. □

Lemma 4.9. *G_{inf} is not DB.*

Proof. The grammar generates infinitely many non-branching derivation trees for the string b, thus generating arbitrarily deep derivation trees whose frontier consists of only one symbol. □

Lemma 4.10. *G_{inf} is not OLP_S.*

Proof. The grammar's language is $\{b\}$; there exist infinitely many derivation trees for the string b of arbitrary depths, each consisting of a non-branching dominance chain in which all nodes have the same yield, therefore there exists no finite-ranged function mapping each two nodes dominating the same yield on every derivation tree to a different image. □

The grammar G_{DB} of figure 2.11 (page 19).

Lemma 4.11. *G_{DB} is not OLP_{PW}.*

Proof. Figure 4.4 lists the extracted context-free backbone of G_{DB}. A context-free backbone containing the rule $P \longrightarrow P$ can generate a unary branching chain of P's which immediately leads to infinite ambiguity. Therefore G_{DB} is not an OLP_{PW} grammar.

$$\mathcal{R} = \left\{ \begin{array}{l} S \longrightarrow P \\ P \longrightarrow P\,Q \\ P \longrightarrow P \\ P \longrightarrow Q \end{array} \right\}$$

Figure 4.4: The context-free backbone of the grammar G_{DB} of figure 2.11.

□

Lemma 4.12. *G_{DB} is neither OLP_{JO_1} nor OLP_{JO_2}.*

Proof. The grammar's language is $\{b^+\}$; a string of l occurrences of b has just one parse tree and its depth is 2^l. In order to generate the string b^l for $l > 1$, the second rule must be applied at least once, generating a lexical item and refilling the innerCount list, and then in order to generate the next lexical item, there must be $|\text{innerCount}|$ applications of the third rule until innerCount list is empty, resulting in a non-branching dominance chain of P's. Therefore G_{DB} is neither OLP_{JO_1} nor OLP_{JO_2}. □

Lemma 4.13. *G_{DB} is DB and FA.*

Proof. A string of l occurrences of b has just one parse tree, and its depth is 2^l; the depth of a tree generating a string of $l+1$ occurrence of b is 2^{l+1}; between the generation of the l^{th} and $(l+1)^{th}$ b's there must be 2^l derivation steps of sentential forms of length l. Therefore, the depth of any derivation tree for a sentential form of length l is bounded by an exponential function of l, and the grammar is depth-bounded and finitely ambiguous. □

Lemma 4.14. *G_{DB} is not OLP_S.*

Proof. By lemma 2.4, in order to generate the string b^l exactly 2^l derivation steps must be applied (the last b is generated on the $2^l th$ derivation step); in order to generate b^{l+1} exactly 2^{l+1} derivation steps must be applied. Between the generation of the l^{th} and $(l+1)^{th}$ symbols there must be 2^l derivation steps of nodes all dominating the same yield. Assume towards a contradiction that there exists a finite-ranged function of range k, $k \leq 2^l$, mapping each two nodes on the non-branching dominance chain to a different image. Then in order to generate the string b^{l+2} the function must map at least two such nodes to the same image for every natural number l. Therefore, there exist no finite-ranged mapping function satisfying the conditions and the grammar is not OLP_S. □

Lemma 4.15. *G_{DB} is not HP.*

Proof. The grammar's language is $\{b+\}$, for every string of l occurrences of b there exists exactly one derivation tree and its depth is 2^l, therefore not every $w \in L(G_{DB})$ has a polynomial (in $|w|$) depth derivation tree. □

4.2 The relationships between the OLP variants

OLP_{PW} VS. OLP_{JO}.

Proposition 4.16. *An OLP_{JO} grammar G is not necessarily an OLP_{PW} grammar.*

Proof. Figure 4.5 lists a skeletal unification grammar, $G_{J\backslash PW}$, generating the language $\{a, b\}$. An a is generated when the category is P and the value of the feature STR is ta. A b is generated when the category is Q and the value of the feature STR is tb. The grammar contains two initial rules (the first and the third). After applying the first rule, the only applicable rule is the second rule, generating a b, then no other rule may be applied. After applying the third rule, the only applicable rule is the fourth rule, generating an a, then no other rule may be applied. Therefore, the grammar can only generate the two derivation trees shown in the figure. Since both derivations are OLP_{JO} derivations, the grammar is both OLP_{JO_1} and OLP_{JO_2}. $G_{J\backslash PW}$ is not OLP_{PW}; figure 4.6 lists the extracted context-free backbone of the grammar. The context-free backbone can generate a unary branching chain of $P - Q - P$, and therefore it is infinitely ambiguous. □

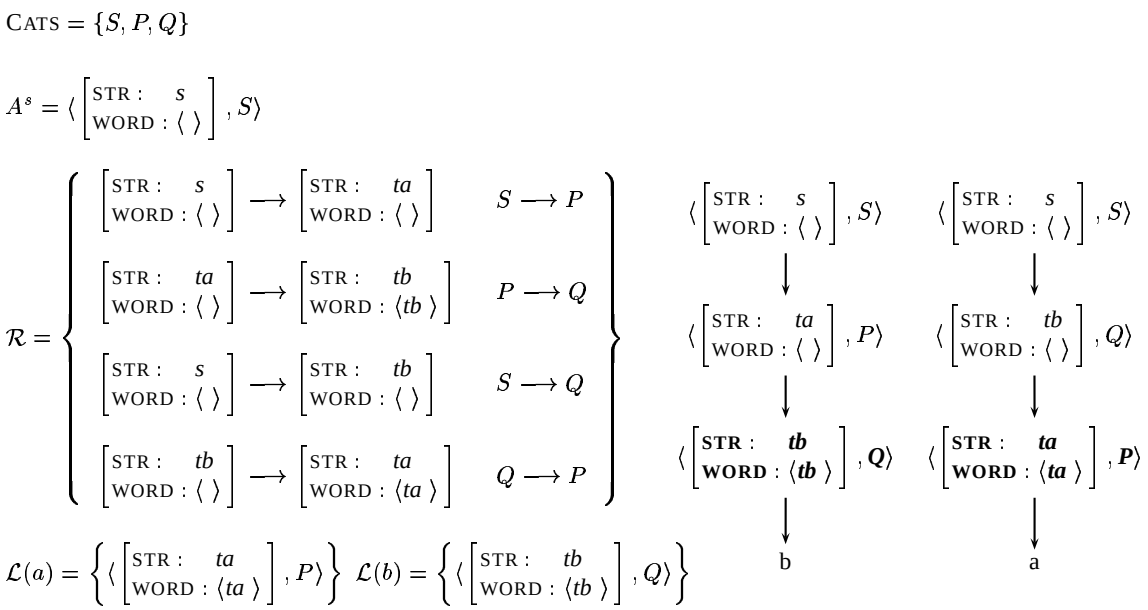


Figure 4.5: An OLP_{JO} grammar $G_{J\backslash PW}$, $L(G_{J\backslash PW}) = \{a, b\}$.

Proposition 4.17. *an OLP_{PW} grammar G is not necessarily an OLP_{JO} grammar.*

Proof. The OLP_{PW} definition does not impose any explicit conditions on ϵ's. Figure 4.7 lists a counter example, an OLP_{PW} grammar G_ϵ which is neither OLP_{JO_1} nor OLP_{JO_1}, and an example derivation tree (note that a context-free grammar can be viewed as a skeletal grammar with empty feature structures). G_ϵ is an OLP_{PW} grammar, its context-free backbone is $P \rightarrow Q\ R$. It is easy to verify that the derivation example given by the figure is the only possible derivation tree admitted by G_ϵ. Since every derivation tree's yield contains an ϵ, G_ϵ is not an OLP_{JO} grammar. □

$$\mathcal{R} = \left\{ \begin{array}{l} S \longrightarrow P \\ S \longrightarrow Q \\ P \longrightarrow Q \\ Q \longrightarrow P \end{array} \right\}$$

Figure 4.6: The context-free backbone of the grammar of figure 4.5.

$$\begin{array}{l} P \rightarrow Q\ R \\ Q \rightarrow b \\ R \rightarrow \epsilon \\ \\ L(G_\epsilon) = \{b\} \end{array} \qquad \begin{array}{ccc} & P & \\ \swarrow & & \searrow \\ Q & & R \\ \downarrow & & \downarrow \\ b & & \epsilon \end{array}$$

Figure 4.7: An example OLP_{PW} grammar, G_ϵ.

$OLP_{PW} \quad VS. \quad Depth - Boundedness$.

Proposition 4.18. *A DB grammar G is not necessarily an OLP_{PW} grammar.*

Proof. By lemma 4.13, the grammar G_{DB} of figure 2.11 is depth-bounded. Figure 4.4 lists the extracted context-free backbone of G_{DB}. The context-free backbone can generate a unary branching chain of P's which immediately leads to infinite ambiguity. □

Proposition 4.19. *If G is OLP_{PW} then G is DB.*

Proof. Let G be an OLP_{PW} grammar. By corollary 3.2, the depth of any derivation tree generated by G is bounded by a linear function of the size of the derived string. Therefore, for every sentential form of length l every derivation tree's depth is bounded by a function of l. □

$OLP_{PW} \quad VS. \quad FiniteAmbiguity$.

Proposition 4.20. *An FA grammar G is not necessarily an OLP_{PW} grammar.*

Proof. By lemma 4.13, the grammar G_{DB} of figure 2.11 is finitely ambiguous. Figure 4.4 lists the extracted context-free backbone of G_{DB}. The context-free backbone can generate a unary branching chain of P's which immediately leads to infinite ambiguity. □

Proposition 4.21. *If G is OLP_{PW} then G is FA.*

Proof. Finite ambiguity is an extension OLP_{PW} to general unification grammars. Therefore any OLP_{PW} grammar is finitely ambiguous. □

OLP_{PW} VS. OLP_S.

Proposition 4.22. *An OLP_S grammar G is not necessarily an OLP_{PW} grammar.*

Proof. Figure 4.8 lists an example OLP_S grammar and a derivation tree. It is easy to verify that the given derivation is the only possible derivation admitted by G_{S_1}. The derivation is of depth 2, therefore there exists a finite-ranged mapping function satisfying the OLP_S conditions. The grammar is not OLP_{PW}, as its context-free backbone can generate a unary branching chain of S's which immediately leads to infinite ambiguity.

$$\textsc{Cats} = \{S\}$$

$$A^s = \langle \left[\textsc{f} : s \right], S \rangle$$

$$\mathcal{R} = \left\{ \left[\textsc{f} : \boxed{1} \right] \longrightarrow \left[\textsc{f} : \left[\textsc{f} : \boxed{1} \right] \right] \qquad S \longrightarrow S \right\}$$

$$\mathcal{L}(b) = \left\{ \langle \left[\textsc{f} : \left[\textsc{f} : s \right] \right], S \rangle \right\}$$

$\langle \left[\textsc{f} : s \right], S \rangle$

$\downarrow$

$\langle \left[\textsc{f} : \left[\textsc{f} : s \right] \right], S \rangle$

$\downarrow$

b

Figure 4.8: An example OLP_S grammar, G_{S_1}.

□

Proposition 4.23. *If G is OLP_{PW} then G is OLP_S*

Proof. By lemma 3.1, if G is OLP_{PW} then it cannot generate a derivation tree in which two nodes have the same category and span the same yield. Therefore there exists a finite ranged function F, whose range is of size $|\textsc{Cats}|$, mapping each feature structure on the derivation tree to its category value,

$$\forall X \in \textsc{Cats} : F\left(\begin{bmatrix} \textsc{cat} : X \\ \vdots \end{bmatrix} \right) = \left[\textsc{cat} : X \right].$$

Thus for every A, $F(A) \sqsubseteq A$ and it is guaranteed that, in every derivation tree, every two nodes spanning the same yield are mapped by F to a different image. □

OLP_{PW} VS. $HonestParsability$.

Proposition 4.24. *An HP grammar G is not necessarily an OLP_{PW} grammar.*

Proof. By lemma 4.7, the grammar G_{inf} of figure 2.8 is an HP grammar. The grammar's context-free backbone, listed in figure 4.3, contains the rules $P \rightarrow P$ and $Q \rightarrow Q$, and therefore G_{inf} is infinitely ambiguous. □

Proposition 4.25. *If G is OLP_{PW} then G is HP.*

Proof. Let G be an OLP_{PW} grammar. By corollary 3.2, the depth of any derivation tree generated by G is bounded by a linear function of the size of the derived string. Therefore, for every $w \in L(G)$ there exists a derivation tree of less than a polynomial depth. □

OLP_{JO} *VS.* $Depth - Boundedness$.

Proposition 4.26. *If G is OLP_{JO_1} then G is DB.*

Proof. An OLP_{JO_1} grammar G can only generate OLP_{JO} derivation trees. By lemma 3.3, any OLP_{JO} derivation tree is of a linear depth in the size of its yield. Therefore, for every sentential form of length n, every derivation tree's depth is bounded by a function of n. □

Proposition 4.27. *An OLP_{JO_2} grammar G is not necessarily a DB grammar.*

Proof. An OLP_{JO_2} grammar requires that for every $w \in L(G)$ there **exists** an OLP_{JO} derivation tree. In contrast, a DB grammar requires that for every $w \in L(G)$ **every** derivation is bounded by a function of $|w|$.

Having an OLP_{JO} derivation tree does not necessarily imply that every derivation tree generating a given string is of a bounded depth by a function of the size of its yield. As a counter example, consider the grammar G_{inf} of figure 2.8. The grammar's language is $\{b\}$, there exists an OLP_{JO} derivation for the string b (as shown in figure 4.2), but by lemma 4.9 the grammar G_{inf} is not a DB grammar; there exists no function of the size of the yield bounding each derivation tree's depth. □

Proposition 4.28. *A DB grammar G is not necessarily an OLP_{JO} grammar.*

Proof. A DB grammar G may generate non-OLP_{JO} derivation trees. Furthermore, there may exist some $w \in L(G)$ for which there exists no OLP_{JO} derivation. By lemma 4.13, the grammar G_{DB} of figure 2.11 is a depth-bounded grammar, but by lemma 4.12, G_{DB} is neither OLP_{JO_1} nor OLP_{JO_2}. □

OLP_{JO} *VS.* $FiniteAmbiguity$.

Proposition 4.29. *If G is OLP_{JO_1} then G is FA.*

Proof. By lemma 3.3, the depth of any OLP_{JO} derivation tree is bounded by a linear function of the size of its yield. For a given string, there exist only a finite number of derivation trees up to some bounded depth (in the size of the string). Therefore for every string w there exist a finite number of derivation trees. □

Proposition 4.30. *An OLP_{JO_2} grammar G is not necessarily an FA grammar.*

Proof. A string that has an OLP_{JO} derivation tree may still have an infinite number of non-OLP_{JO} derivations. Figure 4.2 lists an example OLP_{JO} derivation tree for the grammar G_{inf} of figure 2.8. By lemma 4.8, the grammar is infinitely ambiguous; there exist infinitely many derivation trees for the string b. □

Proposition 4.31. *An FA grammar G is not necessarily an OLP_{JO} grammar.*

Proof. By lemma 4.3, the grammar G_{FA} of figure 2.5 is FA. Any string has a unique derivation tree, but by lemma 4.2, G_{FA} is neither OLP_{JO_1} nor OLP_{JO_2}. □

OLP_{JO} *VS.* OLP_S.

Proposition 4.32. *An OLP_{JO_1} grammar G is OLP_S.*

Proof. Since G is OLP_{JO_1} it cannot generate derivation trees in which the same category appears twice in a non-branching dominance chain. Therefore, there exists a finite ranged function F, whose range is of size $|\textsc{Cats}|$, mapping each feature structure on the derivation tree to its category value,

$$\forall X \in \textsc{Cats} : F\left(\begin{bmatrix}\textsc{cat} : X \\ \vdots\end{bmatrix}\right) = \begin{bmatrix}\textsc{cat} : X\end{bmatrix}.$$

Thus for every A, $F(A) \sqsubseteq A$ and it is guaranteed that, in every derivation tree, every two nodes spanning the same yield are mapped by F to a different image. □

Proposition 4.33. *An OLP_{JO_2} grammar G is not necessarily an OLP_S grammar.*

Proof. By lemma 4.6, the grammar G_{inf} of figure 2.8 is OLP_{JO_2}, there exist an OLP_{JO} derivation tree for every $w \in L(G_{inf})$, but by lemma 4.10, the grammar is not OLP_S since there exists no finite-ranged mapping function satisfying the OLP_S conditions. □

Proposition 4.34. *An OLP_S grammar G is not necessarily an OLP_{JO} grammar.*

Proof. The grammar G_{S_1} of figure 4.8 is OLP_S. The grammar is not OLP_{JO} (by both definitions) since its only derivation tree consists of a non-branching dominance chain in which the category S appears twice. □

OLP_{JO} *VS.* $HonestParsability$.

Proposition 4.35. *If G is OLP_{JO} then G is HP.*

Proof. If G is OLP_{JO}, then for every $w \in L(G)$ there exists at least one OLP_{JO} derivation. By lemma 3.3, the derivation's depth is bounded by a linear function of the length of w, therefore, for every $w \in L(G)$ there exists a polynomial function of $|w|$ bounding the derivation's depth. □

Proposition 4.36. *An HP grammar G is not necessarily an OLP_{JO} grammar.*

Proof. Since the honest parsability constraint is not a syntactic property of grammars, the other direction is not necessarily true. Given an HP grammar G, for every $w \in L(G)$ there exists a derivation of a polynomial depth, but not every w necessarily has an OLP_{JO} derivation. By lemma 4.3, the grammar G_{FA} of figure 2.5 is an HP grammar, but by lemma 4.2, G_{FA} is not OLP_{JO}. □

$Depth - Boundedness \quad VS. \quad FiniteAmbiguity$.

Proposition 4.37. *If G is DB then G is FA.*

Proof. G is DB, therefore there exists a function bounding each derivation tree's depth in the size of its yield. Since there is only a finite set of rules, only a finite number of derivation trees may be generated up to some bounded depth. Therefore every w has a finite number of derivation trees and G is an FA grammar. □

Proposition 4.38. *An FA grammar G is not necessarily a DB grammar.*

Proof. As a counter example we show an FA grammar which is not a DB grammar. Haas (1989) provides an example grammar which is finitely ambiguous but is not depth-bounded. The grammar G_{FA} of figure 2.5 is a general unification grammar variation of his grammar example. By lemma 4.3, the grammar is finitely ambiguous; the grammar induces a finite number of derivation trees on every string. The grammar is not depth-bounded; Haas allows partial derivation trees with non-terminals at their leaves, therefore the second rule may be applied repeatedly many times, generating arbitrarily deep parse trees whose frontier is a non-terminal of length 1. □

$Depth - Boundedness \quad VS. \quad OLP_S$.

Proposition 4.39. *An OLP_S grammar G is not necessarily a DB grammar.*

Proof. The grammar G_{S_1} of figure 4.8 is OLP_S. The grammar is not depth-bounded, as its sole rule may be applied repeatedly many times, resulting in arbitrarily deep parse trees whose frontier has only one symbol. □

Proposition 4.40. *A DB grammar G is not necessarily a OLP_S grammar.*

Proof. By lemma 4.13, the grammar G_{DB} of figure 2.11 is depth-bounded, but by lemma 4.14, the grammar is not OLP_S. □

$Depth - Boundedness \quad VS. \quad HonestParsability$.

There exist two main differences between the two definitions. HP requires that for every $w \in L(G)$ there **exist** a **polynomial** depth derivation, whereas DB requires that **every** derivation for a sentential form of $|w|$ symbols be of a bounded depth by a function of $|w|$ (not necessarily polynomial).

Proposition 4.41. *A DB grammar G is not necessarily an HP grammar.*

Proof. An HP grammar requires that for every $w \in L(G)$ there **exist** at least one derivation of a polynomial depth in the size of w. Therefore, a grammar for which **every** derivation tree is of an exponential depth is not an HP grammar.

By lemma 4.13, the grammar G_{DB} of figure 2.11 is depth-bounded; every string of length n has a unique parse tree of depth 2^n and there exist no arbitrarily deep partial trees with non-terminals at their leaves.

By lemma 4.15, the grammar is not an HP grammar; for every $w \in L(G_{DB})$ every parse tree is of an exponential depth (in $\mid w \mid$). □

Proposition 4.42. *An HP grammar G is not necessarily a DB grammar.*

Proof. The definition of a DB grammar requires that for every $w \in L(G)$ **every** derivation tree be of a bounded depth in the size of w. Therefore, a grammar for which there **exists** a polynomial depth derivation for every string, but may also generate arbitrarily deep partial derivation trees whose frontier consists of non-terminals, is not a depth-bounded grammar.

By lemma 4.3, the grammar G_{FA} of figure 2.5 is an HP grammar; for every string there exists a derivation tree of a polynomial depth in the size of the string.

By lemma 4.4, the grammar is not a DB grammar; the grammar can generate arbitrarily deep parse trees whose frontier has only one symbol. □

$FiniteAmbiguity \quad VS. \quad OLP_S$.

Proposition 4.43. *If G is OLP_S then G is FA*

Proof. By lemma 3.6, if G is OLP_S then the depth of every derivation tree for a string of length l is bounded by $|range(F)| \times l$ (where F is a finite-ranged function satisfying the constraint). There exist only a finite number of derivations up to a certain depth (using a finite set of rules), therefore, for every w there exist a finite number of derivation trees. □

Proposition 4.44. *An FA grammar G is not necessarily an OLP_S grammar.*

Proof. By lemma 4.3, the grammar G_{FA} of figure 2.5 is an FA grammar, for every $w \in L(G_{FA})$ there exists exactly one derivation tree, but by lemma 4.5, G_{FA} is not OLP_S. □

$FiniteAmbiguity \quad VS. \quad HonestParsability$.

Proposition 4.45. *An HP grammar G is not necessarily an FA grammar.*

Proof. By lemma 4.7, the grammar G_{inf} of figure 2.8 is an HP grammar, but by lemma 4.8, G_{inf} is infinitely ambiguous. □

Proposition 4.46. *An FA grammar G is not necessarily an HP grammar.*

Proof. Any depth-bounded grammar is also finitely ambiguous. Therefore the depth-bounded grammar G_{DB} of figure 2.11 is finitely ambiguous, but by lemma 4.15, the grammar is not HP; each derivation tree is of an exponential depth by the size of the derived string. □

$OLP_S \quad VS. \quad HonestParsability$.

Proposition 4.47. *If G is OLP_S then G is HP.*

Proof. By lemma 3.6, the depth of every OLP_S derivation tree for a string of l symbols is bounded by $|range(F)| \times l$. Since $|range(F)|$ is a constant number (independent of l), for every $w \in L(G)$ there exists a polynomial function in $|w|$ bounding the derivation's depth. □

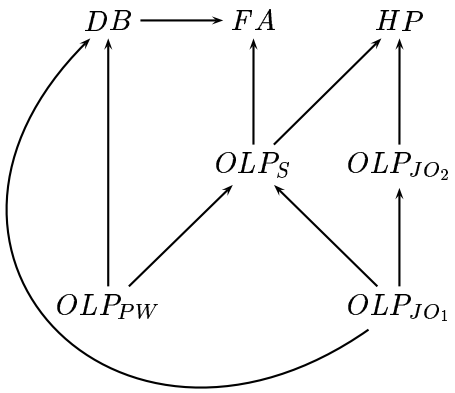
Proposition 4.48. *An HP grammar G is not necessarily an OLP_S grammar.*

Proof. By lemma 4.3, the grammar G_{FA} of figure 2.5 is an HP grammar, but by lemma 4.5, G_{FA} is not an OLP_S grammar. □

4.3 A hierarchy of OLP variants.

Figure 4.9 depicts the inter-relations hierarchy diagram of the OLP variants, separated for skeletal and general unification grammars. The hierarchy is based on the above analysis where the arrows represent the discussed implications; any grammar satisfying a certain restriction also satisfies the targeted one. From the diagram it can be noticed that the OLP variants that apply only to skeletal grammars (except for OLP_{JO_2}) are more restrictive.

Hierarchy for **skeletal** grammars:



Hierarchy for general **unification** grammars:

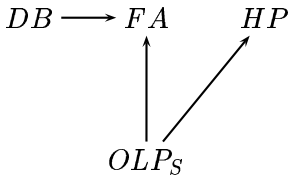


Figure 4.9: Inter-relations Hierarchy diagram.

Chapter 5

Undecidability proofs

Many researchers conjecture that some of the OLP variants are undecidable: it is undecidable whether a grammar satisfies the constraint. However, none of them provides any proof of it. In this chapter we provide proofs of undecidability for three of the undecidable OLP definitions: Finite Ambiguity (FA), Depth-Boundedness (DB) and Shieber's OLP (OLP_S).

5.1 From Turing machines to unification grammars

In order to prove that depth-boundedness and OLP_S are undecidable we use a reduction from the Turing machines halting problem on the empty input to unification grammars; we show that an algorithm that decides depth-boundedness or OLP_S can also solve the universal halting problem. We first define (a variant of) Turing machines below: it is a machine with a single head, a two-way infinite tape and three operations: rewriting a symbol on the tape (without moving the head), a left head move and a right head move (without changing the contents of the tape). The machine accepts an input by a single finite state.

Definition 5.1 (Turing machines). *A (deterministic)* **Turing machine** $(Q, \Sigma, \flat, \delta, s, h)$ *is a tuple such that:*

- Q *is a finite set of states*
- Σ *is a finite alphabet, not containing the symbols* L, R *and* $elist$
- $\flat \in \Sigma$ *is the blank symbol*
- $s \in Q$ *is the initial state*
- $h \in Q$ *is the final state*
- $\delta : (Q \setminus \{h\}) \times \Sigma \to Q \times (\Sigma \cup \{L, R\})$ *is a total function specifying transitions.*

Johnson (1988) proves that the recognition problem is undecidable by showing that every Turing machine can be translated into an *attribute-value grammar* and then by solving the recognition problem, the Turing machines halting problem may also be solved. Francez and Wintner (In preperation) rephrase his proof in terms of feature structures. They show how grammars can

simulate the operation of a Turing machine. They define a unification grammar, G_M, for every Turing machine, M, such that the grammar generates the word halt if and only if the machine accepts the empty input string.

$$L(G_M) = \begin{cases} \{\mathsf{halt}\} & \text{if } M \text{ terminates for the empty input} \\ \emptyset & \text{if } M \text{ does not terminate on the empty input} \end{cases}$$

Below is a short description of their transformation from a Turing machine to a unification grammar. a detailed description can be found in Francez and Wintner (In preperation).

Let $M = (Q, \Sigma, \flat, \delta, s, h)$ be a Turing machine. Define a (skeletal) unification grammar G_M as follows: let FEATS be {*left, right, curr, first, rest*}, ATOMS $= \Sigma \cup \{elist\}$ and CATS $= Q \cup S$ such that $S \notin Q$. There is one lexical item, halt, where $\mathcal{L}(\mathsf{halt}) = \{\langle [\,], h\rangle\}$.

The grammar rules can be divided to four groups. First, an initial rule is defined for every Turing machine:

$$S \longrightarrow \overset{s}{\begin{bmatrix} curr: & \flat \\ right: & elist \\ left: & elist \end{bmatrix}}$$

The rule simulates the initial configuration of a Turing machine that operates on an empty input string: its state is s, the initial state; its tape is empty; and the head points to a blank symbol.

The second group of rules are defined for rewriting transitions. For every q, σ such that $\delta(q, \sigma) = (p, \sigma')$ and $\sigma' \in \Sigma$, the following rule is defined:

$$\overset{q}{\begin{bmatrix} curr: & \sigma \\ right: & X \\ left: & Y \end{bmatrix}} \longrightarrow \overset{p}{\begin{bmatrix} curr: & \sigma' \\ right: & X \\ left: & Y \end{bmatrix}}$$

That is, if the current state is q and the head points to σ, the next state is p and the head points to σ', while the left and the right portions of the tape are not changed.

A third group of rules is defined for right movement of the head. This case is slightly more complicated, as the situation in which the right portion of the tape is empty must be carefully taken care of. For every q, σ such that $\delta(q, \sigma) = (p, R)$ we define two rules, the first for this extreme case and the second for the default case:

$$\overset{q}{\begin{bmatrix} curr: & \sigma \\ right: & elist \\ left: & X \end{bmatrix}} \longrightarrow \overset{p}{\begin{bmatrix} curr: & \flat \\ right: & elist \\ left: & \begin{bmatrix} first: & \sigma \\ rest: & X \end{bmatrix} \end{bmatrix}}$$

$$\overset{q}{\begin{bmatrix} curr: & \sigma \\ right: & \begin{bmatrix} first: & X \\ rest: & Y \end{bmatrix} \\ left: & W \end{bmatrix}} \longrightarrow \overset{p}{\begin{bmatrix} curr: & X \\ right: & Y \\ left: & \begin{bmatrix} first: & \sigma \\ rest: & W \end{bmatrix} \end{bmatrix}}$$

The first rule is only triggered in case the *right* feature of the head, q, has the value *elist* (any other value it can have is bound to be a complex feature structure, and hence incompatible with the atom *elist*). Since the head moves to the right, the contents of the left portion of the tape are shifted left in the next state, p: its first character is the current σ, and its rest is the current states' left portion. Since the right portion of the current tape is empty, in the next configuration the head points to a blank symbol and the right portion of the tape remains empty.

The second rule is only triggered when the *right* feature of q is *not* empty: it must be a list, as its *first* and *rest* features are being considered. The rule shifts the right portion of the tape leftwards: the next configuration's *curr* feature is the first element in the current configuration's right tape; and the rest of the current configuration's right tape becomes the next configuration's *right*.

The last group of rules handle left movements in a symmetric fashion. For every q, σ such that $\delta(q, \sigma) = (p, L)$ we define two rules:

$$\overset{q}{\begin{bmatrix} curr: & \sigma \\ right: & X \\ left: & elist \end{bmatrix}} \longrightarrow \overset{p}{\begin{bmatrix} curr: & \flat \\ right: & \begin{bmatrix} first: & \sigma \\ rest: & X \end{bmatrix} \\ left: & elist \end{bmatrix}}$$

$$\overset{q}{\begin{bmatrix} curr: & \sigma \\ right: & X \\ left: & \begin{bmatrix} first: & Y \\ rest: & W \end{bmatrix} \end{bmatrix}} \longrightarrow \overset{p}{\begin{bmatrix} curr: & Y \\ right: & \begin{bmatrix} first: & \sigma \\ rest: & X \end{bmatrix} \\ left: & W \end{bmatrix}}$$

G_M can only generate unary branching derivation trees, since it contains only unit-rules.

Lemma 5.1. *G_M can generate at most one complete derivation tree.*

Proof. The Turing machine transitions function, δ, is a total, deterministic function; δ is defined uniquely for every state and symbol (except the final state h).

While transforming a Turing machine operation into a unification grammar, every state becomes a category and every alphabet symbol becomes an atomic value for the feature $curr$ (representing the current tape cell the Turing machine's head points to). Since δ is total and deterministic and each of G_M's rules (except the first and terminating rules) represents an entry in the transitions function (each rule's head consists of a category and feature structure containing a value for the feature $curr$), given a category and a $curr$ value, they can only unify with a single rule's head, therefore at each derivation step exactly one rule may be applied, until the category h is reached, resulting in a unique possible complete derivation tree. □

Francez and Wintner (In preperation) prove that $\mathsf{halt} \in L(G)$ iff M terminates on the empty input.

5.2 Undecidability of Finite Ambiguity

Theorem 5.2. *Given a grammar G and a string w, it is undecidable whether w has a finite number of derivation trees admitted by G.*

Proof idea. *The proof is by a contradiction. We assume that it is decidable whether G generates a finite number of derivation trees for a string w and use that assumption to decide the membership problem, contradicting Johnson's theorem (Johnson, 1988).*

Proof. Assume towards a contradiction that there exists an algorithm, A, deciding whether w has a finite number of derivation trees admitted by G. Construct an algorithm, B, to decide the membership problem, with B operating as follows.

On input G, w where $G = \langle \mathcal{R}, \mathcal{L}, A^s \rangle$ is a unification grammar and w is a string:

1. Construct $G' = \langle \mathcal{R}', \mathcal{L}', A'^s \rangle$ such that $\mathcal{L}' = \mathcal{L}$, $A'^s = A^s$ and $\mathcal{R}' = \mathcal{R} \cup \{A^s \rightarrow A^s\}$.
2. Run algorithm A on G', w.
3. G' induces a finite number of derivation trees on w iff $w \notin L(G)$.

By the construction of G', since $\mathcal{R} \subset \mathcal{R}'$ and they share the same lexicon and start symbol, any derivation tree generated by G can also be generated by G'.

If G' can only generate a finite number of derivation trees for a string w, then there exist no derivation tree for w admitted by G. Suppose that there exists at least one derivation tree for w admitted by G and thus there exists a derivation tree for w admitted by G'. Therefore, by applying the rule $A^s \rightarrow A^s$, G' can generate an infinite number of tree structures for w, contradicting the algorithm's outcome. Since G cannot generate any derivation tree for w, $w \notin L(G)$.

If G' can generate infinitely many derivation trees for a string w, then there exists at least one derivation tree for w admitted by G' which does not contain any applications of the rule $A^s \rightarrow A^s$ and therefore there exists a derivation tree for w in G. Suppose that there exist no derivation tree for w in G' without any applications of the rule $A^s \rightarrow A^s$ (which immediately implies that there exist no derivation tree for w in G), since the rule $A^s \rightarrow A^s$ only loops over the start symbol, applying it would not result in generating a derivation tree for w. Thus G' can generate no derivation tree for w, contradicting the algorithm's outcome. Since there exists a derivation tree for w in G, $w \in L(G)$. □

Corollary 5.3. *Finite Ambiguity is undecidable.*

Proof. By theorem 5.2, it is undecidable whether G can generate a finite number of derivation trees on a string w. Therefore it is undecidable whether for every w (over Σ, the grammar's terminal symbols) there exist a finite number of derivation trees. □

5.3 Undecidability of Depth-Boundedness

A grammar, G, is depth-bounded if there exists a function, f, such that for every sentential form of length n, the depth of every derivation tree is bounded by $f(n)$. A grammar G is not depth-bounded if for every function f there exists a derivation tree for a sentential form of length n whose depth is greater then $f(n)$.

Haas allows partial derivation trees with non-terminals at their leaves as legal derivation trees. A depth-bounded grammar cannot build an unbounded number of tree structures from a bounded number of symbols (terminals/non-terminals). A grammar generating an infinite number of tree structures for a sentential form of length n is not depth-bounded.

Theorem 5.4. *depth-boundedness is undecidable.*

Proof. Assume towards a contradiction that there exists an algorithm, A, for deciding depth-boundedness, that is, deciding whether there exists a function f bounding each derivation tree's depth for a sentential form of length n by $f(n)$. Construct an algorithm, B, to decide the universal halting problem, which is known to be undecidable, with B operating as follows.

On input $M = (Q, \Sigma, \flat, \delta, s, h)$, a Turing machine:

1. Construct G_M, simulating the operation of M on the empty input, as described above.
2. Run algorithm A on G_M.
3. G_M is depth-bounded iff M terminates on the empty input.

If G_M is depth-bounded then G_M can only generate a finite number of partial (non-branching) derivation trees. One of these derivation trees must end with a terminal. Otherwise, since δ is a total function and the grammar rules simulate δ, at any derivation step there is always a next applicable rule (given a state other than h and a symbol there is always a next configuration, therefore a category and a $curr$ value may always unify with some rule's head). Thus each partial derivation tree's leaf (of the finite set of possible partial derivation trees) may unify with some rule's head, resulting in an infinite number of partial derivation trees. Therefore, G_M generates a complete derivation tree and thus $\mathsf{halt} \in L(G)$.

If G_M is not depth-bounded, then G_M generates infinitely many non-branching partial derivation trees, none of which ends with a terminal. Assume towards a contradiction that G_M can generate a complete derivation tree (which is unique by lemma 5.1). Therefore, since δ is deterministic and no rule's head may unify with category h, G_M may only generate a finite set of partial derivation trees (where each is a sub-tree of the complete derivation tree), in contradiction to the claim that G_M is not depth-bounded. Therefore, G_M may not generate any complete derivation trees and thus $\mathsf{halt} \notin L(G)$.

Then, using Francez and Wintner's (In preperation) reduction, by deciding whether $\mathsf{halt} \in L(G)$ we can decide whether M terminates on the empty input. □

5.4 Undecidability of OLP_S

Theorem 5.5. *OLP_S is undecidable.*

Proof. Assume towards a contradiction that there exists an algorithm, A, for deciding whether a grammar satisfies OLP_S, that is, deciding whether there exists a finite-ranged function F such that $F(A) \sqsubseteq A$ for all A and there are no derivation trees admitted by G in which a node $\langle u \rangle$ dominates a node $\langle v \rangle$, both are roots of sub-trees with an identical yield and $F(u) = F(v)$. Construct an algorithm, B, to decide the universal halting problem, which is known to be undecidable, with B operating as follows.

On input $M = (Q, \Sigma, \flat, \delta, s, h)$, a Turing machine:

1. Construct G_M, simulating the operation of M on the empty input, as described above.
2. Construct G'_M by adding the rule $A^s \to A^s$ to G_M's set of rules: $\mathcal{R}'_M = \mathcal{R}_M \cup \{A^s \to A^s\}$.
3. Run algorithm A on G'_M.
4. G'_M is OLP_S iff M does not terminate on the empty input.

If G'_M is OLP_S then there exists a finite-ranged function mapping each two descendant nodes sharing the same yield to a different image, therefore there exist no derivation tree for halt admitted by G'_M. Suppose that there exists a derivation tree for halt admitted by G'_M therefore, by applying the rule $A^s \to A^s$, G'_M can generate infinitely many derivation trees for the string halt of arbitrary depths, thus there exist no finite-ranged function mapping each two nodes on a derivation with the same yield to a different image, contradicting the algorithm's outcome. Therefore, there exists no derivation tree for halt admitted by G_M, $\mathsf{halt} \notin L(G_M)$.

If G'_M is not OLP_S, then every finite-ranged function must map at least two descendant nodes sharing the same yield on a derivation tree to the same image, therefore there exists a derivation tree for halt admitted by G'_M. By the construction of G'_M, there exists a derivation tree for halt admitted by G_M, $\mathsf{halt} \in L(G_M)$.

Then, using Francez and Wintner's (In preperation) reduction, by deciding whether $\mathsf{halt} \in L(G)$ one can decide whether M terminates on the empty input. □

Chapter 6

A novel OLP constraint - OLP_D

In this chapter we present our main contribution, a decidable OLP constraint. Our constraint applies to both skeletal and general unification grammars and unlike all the definitions that apply to general unification grammars, it can be tested efficiently; there exists an algorithm for deciding whether a grammar satisfies it. We also provide some improvements to our constraint. The improved constraints are more liberal than all existing decidable constraints.

6.1 A decidable definition of OLP (version 1)

In the following section we present the first version of our OLP constraint and prove that it guarantees decidability of the recognition problem. In this version we assume that ϵ does not appear as a lexical form annotation of any (terminal) node; the grammars contain no ϵ-rules. In the improvements section we provide a more liberal constraint which admits grammars including ϵ-rules.

6.1.1 A decidable OLP constraint, OLP_{D_1}

Definition 6.1. *A sequence of unit-rules $R_1, \ldots, R_k$ ($k \geq 1$) is **cyclicly unifiable** iff there exists a sequence of MRSs $\sigma_1, \ldots, \sigma_{k+2}$ of length 1 (feature structures) such that for $1 \leq i \leq k$, $\sigma_i \rightarrow \sigma_{i+1}$ by the rule R_i, and $\sigma_{k+1} \rightarrow \sigma_{k+2}$ by R_1.*

Figure 6.1 lists two grammar rules, ρ_1, ρ_2. The sequence $\langle \rho_1, \rho_2 \rangle$ is cyclicly unifiable, e.g. by $\langle \sigma_1 = \begin{bmatrix} \text{CAT} : P \\ \text{F} : a \end{bmatrix}, \sigma_2 = \begin{bmatrix} \text{CAT} : Q \\ \text{F} : a \end{bmatrix}, \sigma_3 = \begin{bmatrix} \text{F} : b \end{bmatrix}, \sigma_4 = \begin{bmatrix} \text{CAT} : Q \\ \text{F} : b \end{bmatrix} \rangle$.
σ_1 is unifiable with ρ_1's head, $\sigma_1 \rightarrow \sigma_2$ by ρ_1, $\sigma_2 \rightarrow \sigma_3$ by ρ_2, and then $\sigma_3 \rightarrow \sigma_4$ by ρ_1.

The sequence $\langle \rho_2, \rho_1 \rangle$ is not cyclicly unifiable; whatever ρ_2 applies to, the resulting feature structure is $\begin{bmatrix} \text{F} : b \end{bmatrix}$; then, applying ρ_1 necessarily yields $\begin{bmatrix} \text{CAT} : Q \\ \text{F} : b \end{bmatrix}$, which is incompatible with the head of ρ_2. Hence ρ_2 cannot be applied again.

Definition 6.2 (A decidable OLP constraint (OLP_{D_1})). *A grammar G is off-line parsable iff it contains no cyclicly unifiable sequences.*

$$\mathcal{R} = \left\{ \begin{array}{lll} \rho_1 : & \begin{bmatrix} \text{CAT} : P \\ \text{F} : \boxed{1} \end{bmatrix} & \longrightarrow \begin{bmatrix} \text{CAT} : Q \\ \text{F} : \boxed{1} \end{bmatrix} \\ \rho_2 : & \begin{bmatrix} \text{F} : a \end{bmatrix} & \longrightarrow \begin{bmatrix} \text{F} : b \end{bmatrix} \end{array} \right\}$$

Figure 6.1: An example grammar rules.

Lemma 6.1. *If a grammar G contains no cyclicly unifiable sequences, G does not license any derivation tree with a non-branching dominance chain in which the same rule is used more than once.*

Proof. Assume towards a contradiction that a unit-rule ρ_1 is used more than once in a non-branching dominance chain. Therefore, there exists a sequence of MRSs $\sigma_1, \ldots, \sigma_{k+2}$, the chain nodes on the derivation tree, and a sequence of unit-rules $\rho_1, \ldots, \rho_k$, such that for $1 \leq i \leq k$, $\sigma_i \rightarrow \sigma_{i+1}$ by ρ_i, and $\sigma_{k+1} \rightarrow \sigma_{k+2}$ by ρ_1. Thus, the grammar contains a cyclicly unifiable sequence, a contradiction. □

Lemma 6.2. *The depth of every derivation tree whose yield is of length n admitted by an OLP_{D_1} grammar G is bounded by $u \times n$, where u is the number of G's unit-rules.*

Proof. Since the grammar contains no cyclicly unifiable sequences, by lemma 6.1 no rule may be applied more than once in a non-branching dominance chain. Therefore, the depth of any generated non-branching dominance chain is bounded by u, thus in every derivation tree admitted by G, every u consecutive applications of unit-rules (at most) are followed by either a terminating node or an application of a non-unit-rule expanding the yield (recall that no ϵ-rules are allowed). Therefore, the depth of every derivation tree is at most u times the size of its yield. □

Corollary 6.3. *Parsing termination is guaranteed for OLP_{D_1} grammars.*

Proof. Since the depth of every derivation tree admitted by an OLP_{D_1} grammar is bounded by a linear function of the size of its yield, it is possible to enumerate the derivation trees that have a given string as their yield, therefore parsing termination and decidability of the recognition problem are guaranteed. □

Theorem 6.4. *It is decidable whether a grammar is OLP_{D_1}.*

Proof. In the next section we present an algorithm for deciding OLP_{D_1}. □

6.1.2 An algorithm for deciding OLP_{D_1}

In order to detect cyclicly unifiable sequences, only unit-rules should be considered; we use a graph annotation and search for cycles in the graph.

We first create a *unit-rules transitions graph, UTG,* a directed transitions graph representing unifiability; every vertex is a unit-rule, and an edge leads from u to v iff the body of u is unifiable with the head of v (the body is of length 1). The head and the single element in the body of a unit-rule ρ_i are represented by H_i, B_i respectively.

Then, we look for cycles in the UTG, which may indicate a cyclicly unifiable sequence. For each cycle, we simulate its operation by consecutively applying all its vertices in order to verify whether they form a cyclicly unifiable sequence. Simulation is done by applying the rules according to the order of the sequence, whereas the initial symbol is the empty feature structure.

The cycle edges represent unifiability between the head and body of each two consecutive cycle vertices, but they are not necessarily indicative of a cyclicly unifiable sequence, as it is not guaranteed that after applying several rules, unifiability between the resulting feature structure and the head of the next rule still holds. Simulation of the cycle should be applied beginning each time with a different cycle vertex, as it is possible that by beginning the simulation with some vertex, the cycle's vertices form a cyclicly unifiable sequence, but for others they do not as exemplified by figure 6.1 above.

Note that the fact, that a grammar contains cyclicly unifiable sequences, does not necessarily imply that any of the cycle vertices may ever be reached; it is undecidable whether any of the cycle's rules may ever be applied. Therefore, the constraint may be ruling out grammars for which parsing termination is guaranteed, but it is still a decidable constraint and allows (along with the improvements) more grammars than the existing decidable constraints.

The algorithm is listed in figure 6.2.

6.1.3 Correctness of the algorithm

In order to look for cyclicly unifiable sequences in a grammar G, only unit-rules should be taken into consideration. We first make a transformation from a grammar to a graph annotation, thus we can use some graph algorithms. We then search for cycles in the graph, and check whether any of these cycles' vertices form a cyclicly unifiable sequence.

Lemma 6.5. *If a sequence of unit-rules does not appear as a cycle in the UTG, then it is not cyclicly unifiable.*

Proof. Let $R_1, \ldots, R_k$ be a cyclicly unifiable sequence, let H_i,B_i be the head and body of each R_i respectively. Therefore, there exists a sequence $\sigma_1, \ldots, \sigma_{k+2}$, such that for each $1 \leq i \leq k-1$, $B_i \sqsubseteq \sigma_{i+1}$ and σ_{i+1} is unifiable with H_{i+1}, therefore B_i is unifiable with H_{i+1}. Furthermore, $B_k \sqsubseteq \sigma_{k+1}$, σ_{k+1} is unifiable with H_1, and therefore B_k is unifiable with H_1. Thus $R_1, \ldots, R_k$ represent a cycle in the UTG. Therefore, if a sequence of rules does not form a cycle in the UTG, it is not a cyclicly unifiable sequence. □

Since any cyclicly unifiable sequence is represented as a cycle in the UTG, only cycles of vertices should be considered. Once a cycle is detected, it represents unifiability between every two consecutive vertices; unifiability between the head and body of two consecutive rules. It still does not necessarily imply that the cycle's vertices form a cyclicly unifiable sequence. We simulate the cycle's operation using the function *is_cyclicly_unifiable*, whose input is a sequence of rules (some rotation of the cycle's vertices), beginning each time with a different cycle vertex. The function applies each of the rules consecutively using unification in context (as defined in Francez and Wintner (In preperation, 121-122)), and if one of the rules may not be applied (the resulting feature structure is not unifiable with the rule's head) then it returns **false**; if all rules have been applied successfully, the function returns **true**.

An algorithm for deciding OLP_{D_1}

scan_grammar(G): Boolean

Input: A unification grammar G.
Output: **true** iff G is OLP_{D_1}.

Construct a directed unit-rules transitions graph, UTG, where

Each vertex is a unit-rule $\rho \in \mathcal{R}$ where $|\rho| = 2$.

There exists a directed edge from vertex $\langle H_1, B_1 \rangle$ toward vertex $\langle H_2, B_2 \rangle$ iff B_1 is unifiable with H_2.

For each cycle $C = C_1, \ldots, C_k, C_1$ in UTG:

for i from 0 to $k-1$

Let $V_1 \ldots V_k$ be the cyclic rotation of C, i positions to the right.

If *is_cyclicly_unifiable*$(V_1, \ldots, V_k)$

Return **false**

Return **true**.

is_cyclicly_unifiable($V_1, \ldots, V_k$): Boolean

$FS = [\,]$ /* the most general feature structure */

for i from 1 to k

$V_i = \langle H_i, B_i \rangle$ is the current rule.

if $FS \sqcup H_i$ fails

Return **false**

else $((FS, 1) \sqcup (V_i, 1) = \langle FS', V_i' \rangle)$,where $V_i' = \langle H_i', B_i' \rangle$

$FS = B_i'$ /* unification in context */

if $FS \sqcup H_1$ fails

Return **false**

Return **true**

Figure 6.2: An algorithm for deciding OLP_{D_1}.

Lemma 6.6. *is_cyclicly_unifiable*$(V_1, \ldots, V_k)$ *returns* ***true*** *iff* $V_1, \ldots, V_k$ *is a cyclicly unifiable sequence.*

Proof. If *is_cyclicly_unifiable*$(V_1, \ldots, V_k)$ returns **true**, then all rules $V_1, \ldots, V_k$ have been applied and V_1 may be applied again. The variable FS contains the resulting feature structure after applying each rule. Consider all of FS intermediate values, let FS_i be the value of FS after applying V_i and all its predecessors. Since FS_k is unifiable with V_1, V_1 may be applied again; let FS_{k+1} be the resulting feature structure. Consider the sequence $\langle \sigma_1, \ldots, \sigma_{k+2} \rangle = \langle \langle [\,] \rangle, \langle FS_1 \rangle, \ldots, \langle FS_{k+1} \rangle \rangle$, for $1 \leq i \leq k$, $\sigma_i \rightarrow \sigma_{i+1}$ by R_i, and $\sigma_{k+1} \rightarrow \sigma_{k+2}$ by R_1, there-

fore, by definition $V_1, \ldots, V_k$ is a cyclicly unifiable sequence.

If *is_cyclicly_unifiable*$(V_1, \ldots, V_k)$ returns **false**, then either there exists some rule V_i, whose head is not unifiable with the resulting feature structure FS, or all rules have been applied and the resulting FS is not unifiable with the head of V_1. Assume that after applying some rules, V_j may not be applied. Since the simulation begins with the most general feature structure, the sequence $\langle\langle[\,]\rangle, \langle FS_1\rangle, \ldots, \langle FS_{j-1}\rangle\rangle$ is the most general sequence after applying $V_1, \ldots, V_{j-1}$: for any other sequence $\langle\langle FS'_0\rangle, \langle FS'_1\rangle, \ldots, \langle FS'_{j-1}\rangle\rangle$ such that each $FS'_{i-1} \rightarrow FS'_i$ by V_i, each $FS_i \sqsubseteq FS'_i$. Hence, if FS_{j-1} is not unifiable with V_j's head then neither is FS'_{j-1}, and there exists no sequence of MRSs satisfying the constraint. Therefore $V_1, \ldots, V_k$ is not a cyclicly unifiable sequence. □

Theorem 6.7. *The algorithm returns **true** iff G is OLP_{D_1}.*

Proof. In order to check whether G contains cyclicly unifiable sequences, only unit-rules should be considered. By lemma 6.5, since a cyclicly unifiable sequence is always represented by a cycle in the UTG, all cyclicly unifiable sequences are always detected.

On each cycle, *is_cyclicly_unifiable* is applied from each of the cycle's vertices. By lemma 6.6, the function returns **true** only for cyclicly unifiable sequences. Therefore, if the algorithm returns **true**, then all cycles have been tested and none of their vertices orderings represent a cyclicly unifiable sequence, thus the grammar contains no cyclicly unifiable sequences and is OLP_{D_1}.

If the algorithm returns **false** then *is_cyclicly_unifiable* returned **true** on a set of vertices, by lemma 6.6, this set represents a cyclicly unifiable sequence, thus the grammar contains at least one cyclicly unifiable sequence and is not OLP_{D_1}. □

6.1.4 Evaluation

OLP_{D_1} is applicable to both skeletal and general unification grammars. Ignoring ϵ-rules, it is more liberal than the existing decidable definitions that are limited to skeletal formalisms only, and unlike all definitions that are applicable to general unification grammars, OLP_{D_1} can be tested efficiently.

The grammars G_{ww} of figure 2.2 and G_{abc} of figure 2.4 are OLP_{D_1}; they contain no unit-rules, therefore no non-branching dominance chains can be generated. The grammar $G_{b^{2^n}}$ of figure 2.14 is OLP_{D_1}, as it contains only one unit-rule, a terminating rule, which clearly is not cyclicly unifiable. Thus OLP_{D_1} allows non-context-free grammars.

The class of OLP_{D_1} grammars contains the class of OLP_{JO_1} grammars (the first definition of OLP_{JO} grammars); since an OLP_{JO_1} grammar cannot generate a derivation tree in which the same category appears twice in a non-branching dominance chain, no rule may be applied more than once in a non-branching dominance chain. Thus, any OLP_{JO_1} grammar G is also OLP_{D_1}.

An OLP_{JO_2} grammar (the second definition of OLP_{JO} grammars) G is not necessarily OLP_{D_1}. In an OLP_{JO_2} grammar for every $w \in L(G)$ there exists an OLP derivation tree, but it does not imply that the grammar cannot generate derivation trees in which the same rule may applied more than once in a non-branching dominance chain. The grammar G_{inf} of figure 2.8 is OLP_{JO_2}, since there exists an OLP_{JO} derivation tree for the string b. G_{inf} is not OLP_{D_1}, its second rule may be applied repeatedly infinitely many times, thus the grammar contains cyclicly unifiable sequences.

Since an OLP_{PW} grammar may contain ϵ-rules, an OLP_{PW} grammar G is not necessarily OLP_{D_1}.

OLP_{D_1} admits grammars whose c-structure may contain a non-branching dominance chain in which the same category may appear twice as long as it is generated by a sequence of unit-rules that is not cyclicly unifiable. Furthermore, it does not assume an explicit context-free skeleton. Figure 6.3 lists an example OLP_{D_1} grammar which is neither OLP_{JO} nor OLP_{PW}.

$$\mathcal{R} = \left\{ \begin{array}{ll} \left[\text{F} : s\right] \longrightarrow \left[\text{F} : a\right] & S \longrightarrow P \\ \left[\text{F} : a\right] \longrightarrow \left[\text{F} : b\right] & P \longrightarrow P \end{array} \right\}$$

$$\mathcal{L}(b) = \left\{ \langle \left[\text{F} : b\right], P \rangle \right\}$$

Figure 6.3: An OLP_{D_1} grammar, G_D.

The following discussion shows that neither HP nor DB nor FA imply OLP_{D_1}.

The grammars G_{FA} of figure 2.5, which is HP and FA, and G_{inf} of figure 2.8, which is HP, are not OLP_{D_1}; e.g., by their second rule and the following set of feature structures:

$$\left\{ \begin{bmatrix} \text{CAT} : & p \\ \text{WORD} : & \langle tb \rangle \end{bmatrix}, \begin{bmatrix} \text{CAT} : & p \\ \text{WORD} : & \langle tb, tb \rangle \end{bmatrix}, \begin{bmatrix} \text{CAT} : & p \\ \text{WORD} : & \langle tb, tb, tb \rangle \end{bmatrix} \right\}$$

Their second rule may be applied repeatedly many times, resulting in a non-branching dominance chain in which the same rule may be applied more than once.

The grammar G_{DB} of figure 2.11 is DB and FA, but it is not OLP_{D_1}; e.g., by the third rule and the following set of feature structures:

$$\left\{ \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb, tb, tb \rangle \\ innerCount : & \langle tb, tb, tb \rangle \end{bmatrix}, \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb, tb, tb, tb \rangle \\ innerCount : & \langle tb, tb \rangle \end{bmatrix}, \begin{bmatrix} \text{CAT} : & p \\ depthCount : & \langle tb, tb, tb, tb, tb \rangle \\ innerCount : & \langle tb \rangle \end{bmatrix} \right\}$$

It can generate a non-branching dominance chain in which the third rule may be applied more than once.

Every OLP_{D_1} grammar G is also HP; by lemma 6.2, the depth of every derivation tree for a string of n symbols is bounded by a linear function of n, therefore for every $w \in L(G)$ there exists a derivation tree whose depth is polynomial in the size of w. For the same reason, an OLP_{D_1} grammar G is also DB. Thus, by the transitivity of implicature, An OLP_{D_1} grammar G is also FA.

An OLP_S grammar G is not necessarily OLP_{D_1}. Figure 6.4 lists an OLP_S grammar generating the language $\{b^+\}$. A string of n occurrences of b has a derivation tree of depth $3 \times n$. The depth of every non-branching chain is 3, furthermore, there exist only four possible feature structures for each node in every derivation tree admitted by G_S, therefore there exists a finite-ranged function F (e.g., mapping each feature structure to itself) such that no two nodes on a derivation tree spanning the same yield are mapped to the same feature structure. The grammar is not OLP_{D_1}, since the first rule may be applied twice consecutively, resulting in a cyclicly unifiable

sequence. In section 6.2 we present an improvement to OLP_{D_1} which admits G_S.

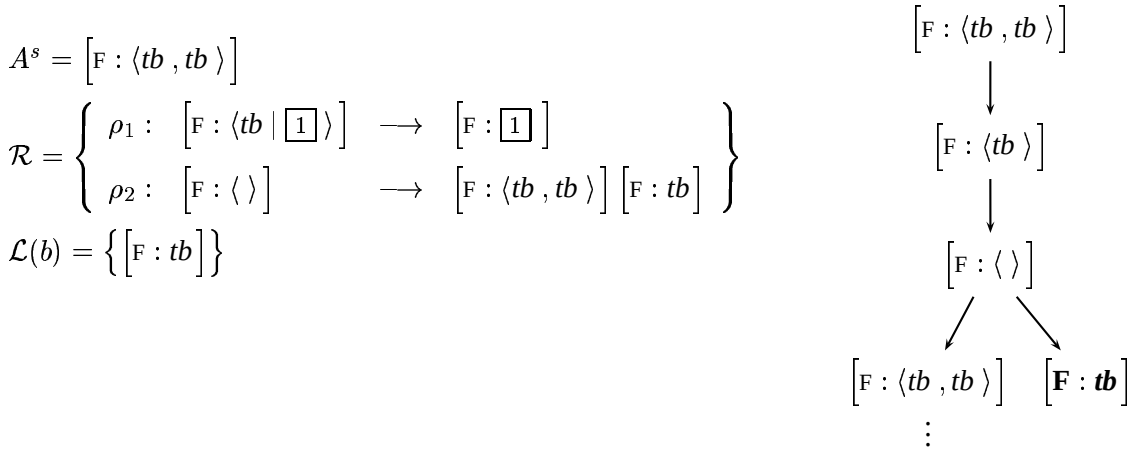


Figure 6.4: An example OLP_S grammar, G_S and its derivation form.

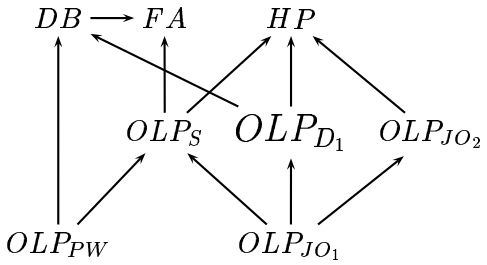
OLP_{D_1} is a restriction on derivation trees such that no two nodes on a derivation tree spanning the same yield are unifiable with the same rule's head. OLP_S is a restriction on derivation trees such that no two nodes on a derivation tree spanning the same yield are mapped to the same image. We tend to believe that an OLP_{D_1} grammar is not necessarily OLP_S, meaning that there exists an OLP_{D_1} grammar for which there exists no finite-ranged mapping function satisfying the OLP_S conditions, but we have not yet been able to come up with one. Our intuition is as follows: by definition in an OLP_S grammar there exists a finite-ranged function f mapping each two nodes on a derivation tree spanning the same yield to a different image. f is finite and maps each feature structure to a certain value, satisfying the constraint, without considering its occurrences on the derivation tree. If a grammar G is OLP_{D_1} then the depth of every sub-derivation dominating the same yield is bounded by the number of G's unit-rules. We conjecture that there might be some cases in which the same feature structure must be mapped to a different image in different chains in order to satisfy the constraint. Thus G is OLP_{D_1} but it might not be OLP_S.

Figure 6.5 depicts the revised inter-relations hierarchy diagram of the OLP definitions including OLP_{D_1}.

Limitations of OLP_{D_1}

The class of OLP_{D_1} grammars can never be equal to any of the other OLP classes for general unification grammars. Since the constraints for general unification grammars are undecidable, if any of these classes were equal to the class of OLP_{D_1}, then using the algorithm for deciding OLP_{D_1}, we could also decide whether a grammar satisfies the other constraint which is undecidable.

Hierarchy for **skeletal** grammars:



Hierarchy for general **unification** grammars:

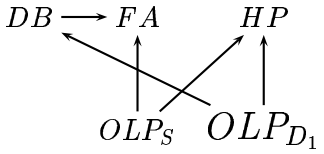


Figure 6.5: Revised hierarchy diagram, OLP_{D_1}.

OLP_{D_1} guarantees parsing termination, but there exist non-OLP_{D_1} grammars for which parsing termination holds. The grammar of figure 6.4 is OLP_S, thus parsing termination is guaranteed but it is not OLP_{D_1}.

Assume that a grammar G contains a cyclicly unifiable sequence, $R_1, \ldots, R_k$. Whether any of $R_1, \ldots, R_k$ may ever be applied in any derivation tree admitted by G is an undecidable problem. Therefore, G is not OLP_{D_1} although parsing termination may still be guaranteed for it.

OLP_{D_1} does not allow any unit-rule sequences in which the same rule may be applied more than once. There might be some unit-rule sequences in which at some point, after applying the sequence rules repeatedly several times, unification between the resulting feature structure and the head of the next rule may no longer hold, hence parsing will terminate. In the next section we propose an improvement of OLP_{D_1} called $OLP_{D\times l}$ which allows such grammars.

6.2 Improvements

6.2.1 A decidable definition of OLP (version 2)

In the following section we present the second version of our OLP constraint. The previous version of our constraint is more liberal than Johnson's constraint, but since it does not permit grammars which contain ϵ-rules, it is incomparable with Pereira and Warren's constraint. Furthermore, ϵ's play a major role in many natural languages descriptions. In this version we allow ϵ-rules and prove that our new constraint is more liberal than the existing decidable constraints.

Let F be the set all rules heads. Let E' be the set consisting of the heads of all rules that may

never derive an ϵ: there exists no sub-derivation tree whose root is an element of E' and whose yield is ϵ. We create a set of *ϵ-derivables*, E, to be the complement of E' consisting of the heads of all the rules that may derive an ϵ: $E = F \setminus E$. We later use the set for defining our constraint.

Definition 6.3 (ϵ-derivables set, E). *Given a grammar $G = \langle \mathcal{R}, \mathcal{L}, A^s \rangle$, let E_d be defined as follows:*

- $E_1 = \{A \mid A \to \epsilon \in \mathcal{R}\}$
- *For $d > 1$, $E_d = \{A \mid A \to A_1, \ldots A_n \in \mathcal{R}$ and there exists a sequence $B_1 \ldots B_n$ such that for each $1 \leq i \leq n$, A_i is unifiable with $B_i \in E_{l_i}$ for some $l_i < d\}$*

Let $E = \bigcup_{1 \leq d \leq |\mathcal{R}|} E_d$.

Lemma 6.8. *E is finite and can be computed in polynomial time (in $|\mathcal{R}|$).*

Proof. E contains at most all rules' heads, therefore the size of E is at most $|\mathcal{R}|$. Each E_i is computed incrementally beginning with E_1. In order to compute E_i, $i > 1$, all rules' bodies should be checked for unifiability with elements of E_d for $1 \leq d < i$. The union of these sets contains at most $|\mathcal{R}|$ elements. Let l be the maximum rule's length, therefore for each rule, at most $(l-1) \times |\mathcal{R}|$ unifiability checks should be made. Thus each E_i can be computed in at most $(l-1) \times |\mathcal{R}|^2$ steps and therefore E is computed in at most $(l-1) \times |\mathcal{R}|^3$ steps ($(l-1) \times |\mathcal{R}|^2 \times d_{max}$). □

Lemma 6.9. *If $A \notin E$, where A is some rule's head, then A may never derive the empty string.*

Proof. Assume towards a contradiction that A may derive the empty string. Therefore there exists a derivation tree of some depth n whose root is A and each of its leaves is ϵ. We next prove that each internal vertex on the derivation tree is unifiable with elements of E. We define the internal vertices *level* as follows: the root is on level $l = n$, all internal vertices on depth $n - i$ are on level $l = i$ (a bottom-up view).

The proof is by induction on l, the tree's level, such that all internal vertices up to level l are unifiable with elements of E. For $l = 1$, since all vertices in level 0 (i.e., the leaves) are ϵ's, all of their mothers are unifiable with ϵ-rule heads. Thus all internal vertices on level 1 are unifiable with elements of E (in fact, of E_1).

Assume that the induction hypothesis holds for all i, $1 \leq i < l$, so that all internal vertices up to level i are unifiable with elements of E. For $i = l$, each node on level l is unifiable with some rule's head, A where $A \to A_1, \ldots, A_m \in \mathcal{R}$. By the induction hypothesis, all of $A_1, \ldots, A_m$ are unifiable with elements of E (since they are on level $l - 1$), therefore, by definition, $A \in E$. Thus each internal node on level l is unifiable with elements of E.

Therefore all daughters of the root A are unifiable with elements of E, hence $A \in E$, a contradiction. □

Definition 6.4. *Given a grammar $G = \langle \mathcal{R}, \mathcal{L}, A^s \rangle$, $UR(G)$ is the following set of rules:*

- *For each rule $\rho \in \mathcal{R}$, if ρ is a unit-rule, then $\rho \in UR(G)$.*
- *For each rule $A \to A_1 \ldots A_n \in \mathcal{R}$, if all of the body elements are unifiable with elements of E, then for $1 \leq i \leq n$, $A \to A_i \in UR(G)$.*

- *For each rule $A \rightarrow A_1 \ldots A_n \in \mathcal{R}$, if all of the body elements but one (A_i) are unifiable with elements of E, then $A \rightarrow A_i \in UR(G)$.*

In this OLP version, as in the previous one, we want to exclude grammars which generate derivation trees in which the same rule may be applied more than once from two different nodes dominating the same yield. Since the grammar may contain ϵ-rules, it is not enough to search for unit-rule chains. The purpose of the second bullet is to prevent multiple applications of the same rule in a sub-derivation tree whose yield is ϵ (for example, the context-free grammar of figure 6.6(a)), thus ruling out grammars generating infinitely deep sub-derivation trees whose yield consists of ϵ's only. The purpose of the third bullet is to consider rules which may generate sub-derivation trees whose yield is of length 1 as unit-rules, thus preventing multiple applications of the same rule in sub-derivation trees dominating the same yield (For example the context-free grammar of figure 6.6(b)).

Definition 6.5 (A decidable OLP constraint (OLP_{D_2})). *A grammar G is off-line parsable iff it contains no cyclicly unifiable sequences of $UR(G)$ rules.*

$P \rightarrow PP$	$P \rightarrow PQ$
$P \rightarrow \epsilon$	$P \rightarrow b$
	$Q \rightarrow \epsilon$
(a)	(b)

Figure 6.6: Motivation for $UR(G)$ rules.

Figure 6.7 lists a grammar example such that,

$$E = \left\{ \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix}, \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \boxed{1} \end{bmatrix}, \begin{bmatrix} \text{CAT}: & s \\ \text{WORD}: & \langle s \rangle \end{bmatrix} \right\}$$

$$UR(G) = \left\{ \begin{array}{ll} \rho_1: & \begin{bmatrix} \text{CAT}: & s \\ \text{WORD}: & \langle s \rangle \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \\ \rho_2: & \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \boxed{1} \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \mid \boxed{1} \rangle \end{bmatrix} \\ \rho_3: & \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \boxed{1} \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \\ \rho_4: & \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \boxed{1} \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \boxed{1} \end{bmatrix} \\ \rho_5: & \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \longrightarrow \epsilon \end{array} \right\}$$

where, ρ_1, ρ_4 and ρ_5 are unit-rules and thus belong to $UR(G)$. ρ_2 and ρ_3 are added to $UR(G)$ by the grammar's second rule, since both its body elements are ϵ-unifiable (the second bullet of definition 6.4).

The sequence $\langle \rho_2 \rangle$ is cyclicly unifiable, for example, by

$$A^s = \begin{bmatrix} \text{CAT}: & s \\ \text{WORD}: & \langle s \rangle \end{bmatrix}$$

$$\mathcal{R} = \left\{ \begin{array}{l} \begin{bmatrix} \text{CAT}: & s \\ \text{WORD}: & \langle s \rangle \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \\ \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \boxed{1} \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \mid \boxed{1} \rangle \end{bmatrix} \quad \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \\ \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \boxed{1} \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \boxed{1} \end{bmatrix} \\ \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \langle tb \mid \boxed{1} \rangle \end{bmatrix} \longrightarrow \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \boxed{1} \end{bmatrix} \quad \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \\ \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \longrightarrow \epsilon \end{array} \right\}$$

$$\mathcal{L}(b) = \left\{ \begin{bmatrix} \text{CAT}: & q \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \right\}$$

Figure 6.7: Cyclicly unifiability example.

$$\left\{ \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix}, \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb, tb \rangle \end{bmatrix}, \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb, tb, tb \rangle \end{bmatrix} \right\}$$

The sequence $\langle \rho_3 \rangle$ is also cyclicly unifiable, for example, by

$$\left\{ \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix}, \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix}, \begin{bmatrix} \text{CAT}: & p \\ \text{WORD}: & \langle tb \rangle \end{bmatrix} \right\}$$

Lemma 6.10. *Let G be an OLP_{D_2} grammar. G does not license any derivation trees in which the same rule is applied more than once from two different nodes dominating the same yield.*

Proof. Assume towards a contradiction that an OLP_{D_2} grammar G can generate a derivation tree which contains a sub-derivation in which the same rule is applied more than once from two nodes dominating the same yield, as shown in figure 6.8. Let A be the dominating feature structure from which the first application of a rule ρ is applied. Let B be its descendant from which ρ may be applied again dominating the same yield.

Such a sub-derivation can result only by consecutive applications of unit-rules, ϵ-deriving rules and rules whose body elements (all but one) derive an ϵ (all other rules are branching rules in which at least two of the body element are non ϵ-derivable, thus expanding the yield).

Let $V_1, \ldots, V_k$ be the applied rules and $FS_1, \ldots, FS_{k-1}$ be the feature structures on the path leading from A to B.

We construct the sequence $V'_1, \ldots, V'_k$ as follows, for each $1 \leq i \leq k$:

- If V_i is a unit-rule then $V'_i = V_i$.
- If $V_i = A \rightarrow A_1, \ldots, A_n$, $A \in E$ and FS_i is the $j-th$ daughter on the derivation tree after applying V_i, then $V'_i = A \rightarrow A_j$.

- If $V_i = A \rightarrow A_1, \ldots, A_n$, and all of the body elements but A_j are unifiable with elements of E, then $V_i' = A \rightarrow A_j$ (the derivation step from FS_{i-1} to FS_i must have been by A_j, otherwise since A_j is non ϵ-derivable, A and B would not share the same yield).

Thus each of $V_1', \ldots, V_k'$ belongs to $UR(G)$ and all of them may be applied consecutively resulting in $FS_1, \ldots, FS_{k-1}, B$. Since V_1 is applied more than once, the head of V_1' is unifiable with B thus V_1' may be applied again resulting in FS. Therefore, the sequence $V_1', \ldots, V_k'$ is cyclicly unifiable by $A, FS_1, \ldots, FS_{k-1}, B, FS$. Hence G contains a cyclicly unifiable sequence and it is not OLP_{D_2}, a contradiction.

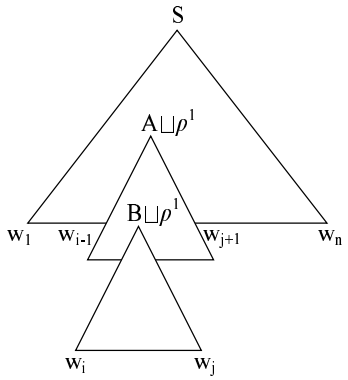


Figure 6.8: An example derivation tree in which A dominates B, both are and unifiable with ρ's head and have the same yield.

□

Lemma 6.11. *The depth of any OLP_{D_2} derivation tree for a string of n symbols is bounded by $|\mathcal{R}| \times n$.*

Proof. Let G be an OLP_{D_2} grammar. By lemma 6.10, the maximum depth of a sub-derivation dominating the same yield is bounded by $|\mathcal{R}|$, thus in every derivation tree after at most $|\mathcal{R}|$ derivation steps, in which all nodes dominate the same yield, there must be either a terminating node or an application of a rule expanding the yield. Therefore, in order to generate a string of n symbols, the depth of every derivation tree is at most $|\mathcal{R}| \times n$. □

Corollary 6.12. *Parsing termination is guaranteed for OLP_{D_2} grammars.*

Proof. Since the depth of every derivation tree admitted by an OLP_{D_2} grammar is bounded by $|\mathcal{R}| \times n$, it is possible to enumerate the derivation trees that have a given string as their yield, therefore parsing termination and decidability of the recognition problem are guaranteed. □

Theorem 6.13. *It is decidable whether a grammar is OLP_{D_2}.*

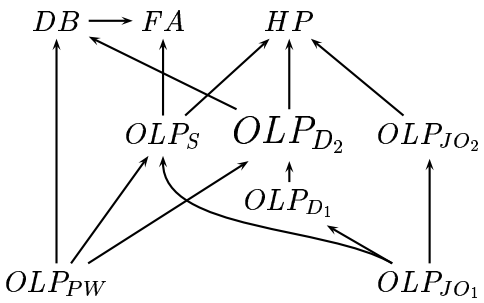
Proof. Since the set $UR(G)$ consists of unit-rules only, the algorithm for deciding OLP_{D_1} of section 6.1.2 can be also used for deciding OLP_{D_2} where each vertex in the UTG graph is a $UR(G)$ rule. □

Evaluation

OLP_{D_2} is more liberal than OLP_{D_1}; since the set of unit-rules is a subset of $UR(G)$, any grammar satisfying OLP_{D_1} would also satisfy OLP_{D_2}. Unlike version 1, any OLP_{PW} grammar G is also OLP_{D_2}; assume towards a contradiction that G contains cyclicly unifiable sequences, thus the same rule may be applied more than once from two different nodes dominating the same yield, resulting in an infinitely ambiguous context-free backbone (by lemma 3.1), a contradiction.

OLP_{D_2} is still not as liberal as the undecidable constraints, but it can be tested efficiently. Figure 6.9 depicts the revised inter-relations hierarchy diagram of the OLP definitions including OLP_{D_2}.

Hierarchy for **skeletal** grammars:



Hierarchy for general **unification** grammars:

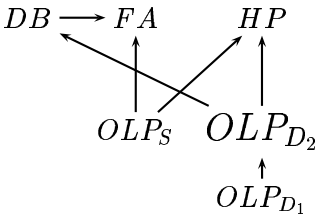


Figure 6.9: Revised hierarchy diagram, OLP_{D_2}.

6.2.2 A decidable definition of OLP (version 3)

We extend the class of OLP_{D_2} grammars by allowing $UR(G)$ rules which may be applied at most a constant number of times. The improved constraint is called $OLP_{D\times l}$, where l is an arbitrary number:

Definition 6.6. *A sequence of $UR(G)$ rules $R_1, \ldots, R_k$ is* l-cyclicly-unifiable *iff $(R_1, \ldots, R_k)^l$ is cyclicly unifiable.*

Definition 6.7. *A grammar G is $OLP_{D\times l}$ iff it contains no l-cyclicly-unifiable sequences.*

The grammar of figure 6.4 is not OLP_{D_2}, but it is $OLP_{D\times 2}$, as its first rule may be applied repeatedly at most twice. Therefore, the sequence $\langle \rho_1, \rho_1 \rangle$ is not cyclicly unifiable.

Parsing termination is guaranteed for $OLP_{D\times l}$ grammars; since the grammar contains no l-cyclicly-unifiable sequences, the depth of any sub-derivation dominating the same yield is bounded by l times the number of grammar rules (where l is a constant number). Therefore, the depth of every derivation tree whose yield is of length n admitted by an $OLP_{D\times l}$ grammar G is bounded by $(l \times |\mathcal{R}|) \times n$.

$OLP_{D\times l}$ is decidable; the algorithm for deciding OLP_{D_1} of figure 6.2 can be extended in order to decide $OLP_{D\times l}$, the only difference, beside using $UR(G)$ instead of the grammar's unit-rules, is that on each cycle *is_cyclicly_unifiable* is called with some cyclic rotation of $(V_1, \ldots, V_k)^l$. *is_cyclicly_unifiable* is unchanged.

As shown above, the grammar of figure 6.4 is not OLP_{D_2}, but it is OLP_S and $OLP_{D\times 2}$. $OLP_{D\times l}$ is an improvement to OLP_{D_2}, but it is still not equivalent to the class of OLP_S grammars; a grammar G is OLP_S if there exists a finite ranged function satisfying the constraint, meaning there exists some l such that $|range(F)| = l$. Given a natural number l it is decidable whether G is $OLP_{D\times l}$. But the question whether there exists some value of l such that G is $OLP_{D\times l}$ is undecidable.

Chapter 7

Conclusions

In this research we deal with a constraint, generally known as the off-line parsability constraint (OLP), for guaranteeing decidability of the recognition problem and parsing termination of unification grammars. We have examined several variants of the OLP constraint and used some grammar examples to emphasize the differences between the OLP variants. We have made a comparative analysis of the several different variants for the first time and presented a hierarchy among them in terms of the classes of grammars each variant allows.

Several researchers (Haas, 1989; Torenvliet and Trautwein, 1995) conjecture that OLP is undecidable, although none of them provides any proof of it. In chapter 5, we provide one of our main contributions, undecidability proofs for three of the undecidable OLP variants: Finite Ambiguity, Depth-Boundedness and Shieber's constraint. We use a reduction from the Turing machines halting problem on the empty input to a unification grammar and show that by deciding whether a grammar is OLP we are also able to decide whether a Turing machine M terminates on the empty input ϵ.

In chapter 6 we provide our main contribution, a novel OLP constraint which is decidable. Our constraint is applicable to both skeletal and general unification grammars. It is more liberal than the existing decidable constraints and unlike all definitions that are applicable to general unification grammars it can be tested efficiently. Along with our constraint we also provide an algorithm for deciding whether a grammar satisfies the constraint, as well as an evaluation of our constraint compared with the other OLP variants. At the end of the chapter, we also suggest some improvements and optimizations to our constraint.

As for future work, while comparing our OLP constraint to Shieber's constraint, we have proved that a grammar satisfying Shieber's constraint does not necessarily satisfy our constraint, but we have not been able to prove the other direction. We conjecture that since these two conditions impose unrelated restrictions, they are incomparable; neither of the classes of grammars they permit contains the other, meaning there might also be some grammar satisfying our constraint which does not satisfy Shieber's constraint, but, so far, we have not been able to come up with one.

There exist some OLP variants which are applicable only to unification grammar formalisms which assume an explicit context-free backbone. General unification grammars do not necessarily yield an explicit non-trivial context-free backbone. As seen in the grammar examples of chapter 2,

although the grammars have no explicit context-free backbone, such a backbone can be deduced through the feature CAT. As for future work, we intend to thoroughly explore these skeletons and try to devise an algorithm for extracting the most informative skeleton when it is not explicit.

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